The role of embodiment in developing higher-order mathematical thinking in primary education

Michiel Veldhuis & Marja van den Heuvel-Panhuizen

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Universiteit Utrecht

Interview with fashion designer
Iris van Herpen
in de Volkskrant, October 28, 2017

The form comes into existence during the process of folding, while her head and hands are working together and at the same time: “A part of the design process is unconscious, I literally let my hands do the work. Because I have been doing this for such a long time, knowledge and intuition went into my hands that go further than what I can consciously think of.” The body saves, according to her, like in dancers and musicians, knowledge through experience and repetition. “This leads me to quickly understand the material and know what I can do with it. It is completely in my hands.”

The role of embodiment in developing higher-order mathematical thinking in primary education

Beyond FLATLAND in primary school mathematics education

dynamic data modeling

probability

early algebra

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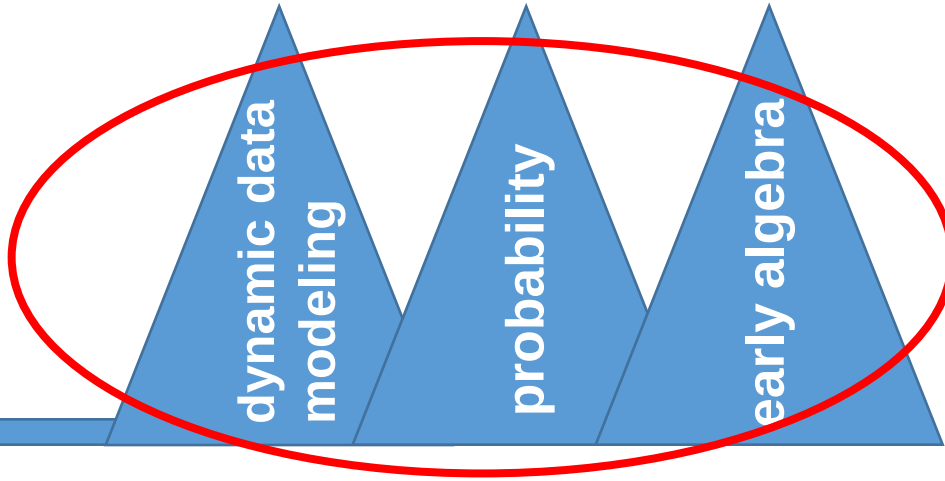
- The role of embodiment in developing mathematical higher-order thinking

Beyond FLATLAND in primary school mathematics education



Develop a **teaching sequence** of six lessons

Learning facilitators:
- Embodied cognition
- Representational redescription theory

Sequencing of tasks:
- Variation theory

Data:
- Videos of students working in the lessons
- Students' written work in the lessons
- Students' responses on lesson-specific test items

- The role of embodiment in developing mathematical higher-order thinking

early algebra

Background

In the Netherlands, teaching algebra starts in the first grades of secondary school (12-13 years)

International research provides evidence that (early) algebra can be taught in primary school (e.g., Brizuela & Schliemann, 2004; Kaput et al., 2008)

Research question

How can primary school students' algebraic reasoning be fostered?

early algebra

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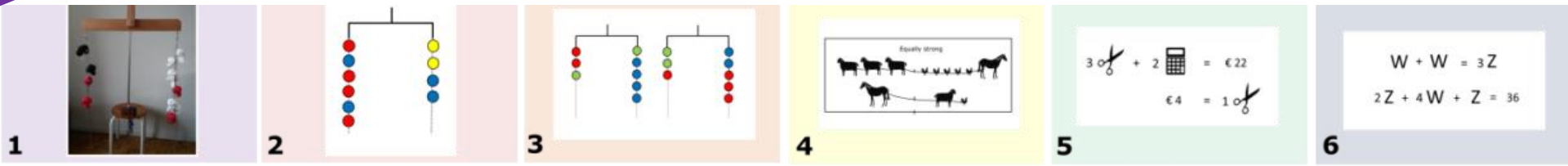
How can primary school students' algebraic reasoning be fostered?



Teaching sequence of six lessons

Focus on:	*Algebraic reasoning with linear equations*
More specifically:	*Reasoning with, and about, unknowns using algebraic strategies*
Context:	*Working with a hanging mobile*
Embodiment/grounding:	*Experience of balance - equivalence*

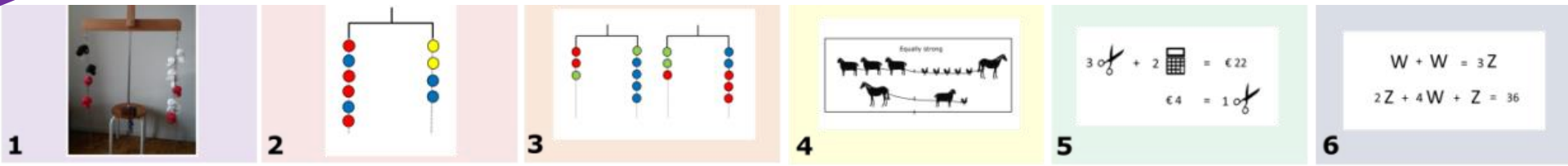
- The role of embodiment in developing mathematical higher-order thinking

early algebra

Research question

Informal → Formal

ing be fostered?



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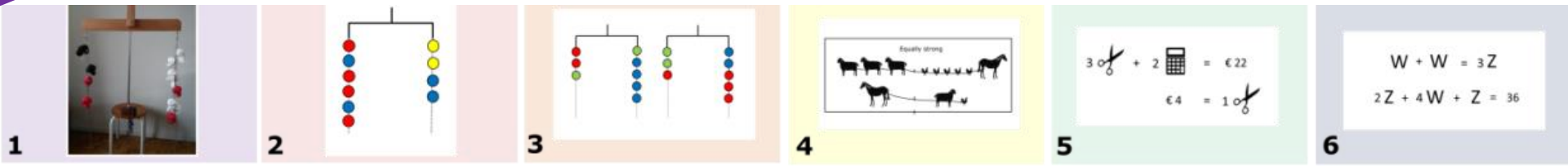

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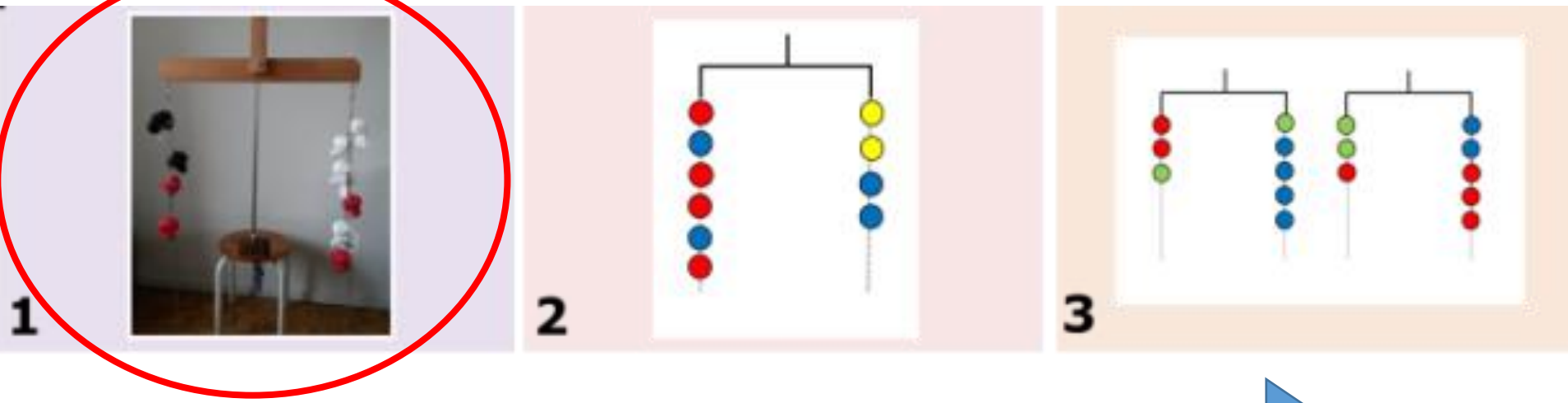
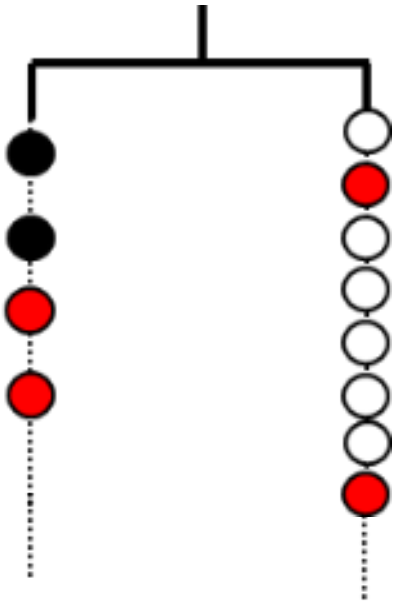


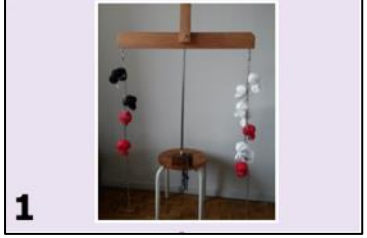
Context → Model

- The role of embodiment in developing mathematical higher-order thinking

early algebra



What can you do to keep the hanging mobile straight?



- **Reactivating the concept of equivalence**
- **Eliciting algebraic strategies**
  - Restructuring
    - Changing sides
    - Changing order of bags on the same side
  - Isolation
    - Taking away similar bags on both sides
    - Taking away different bags on both sides
  - Substitution
    - Replacing bags by bags of another color

early algebra

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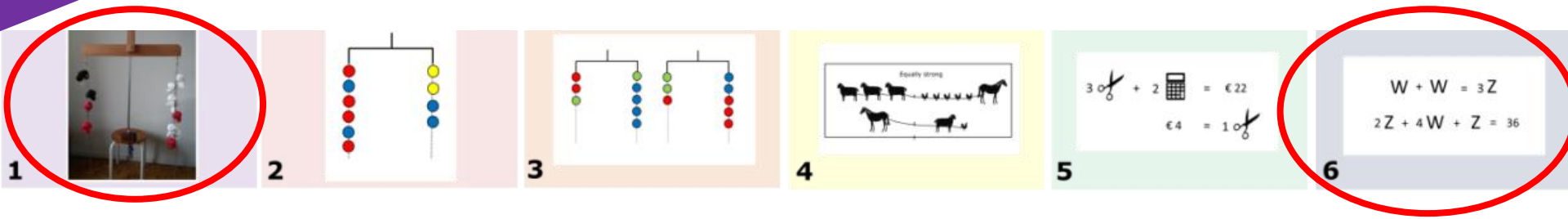

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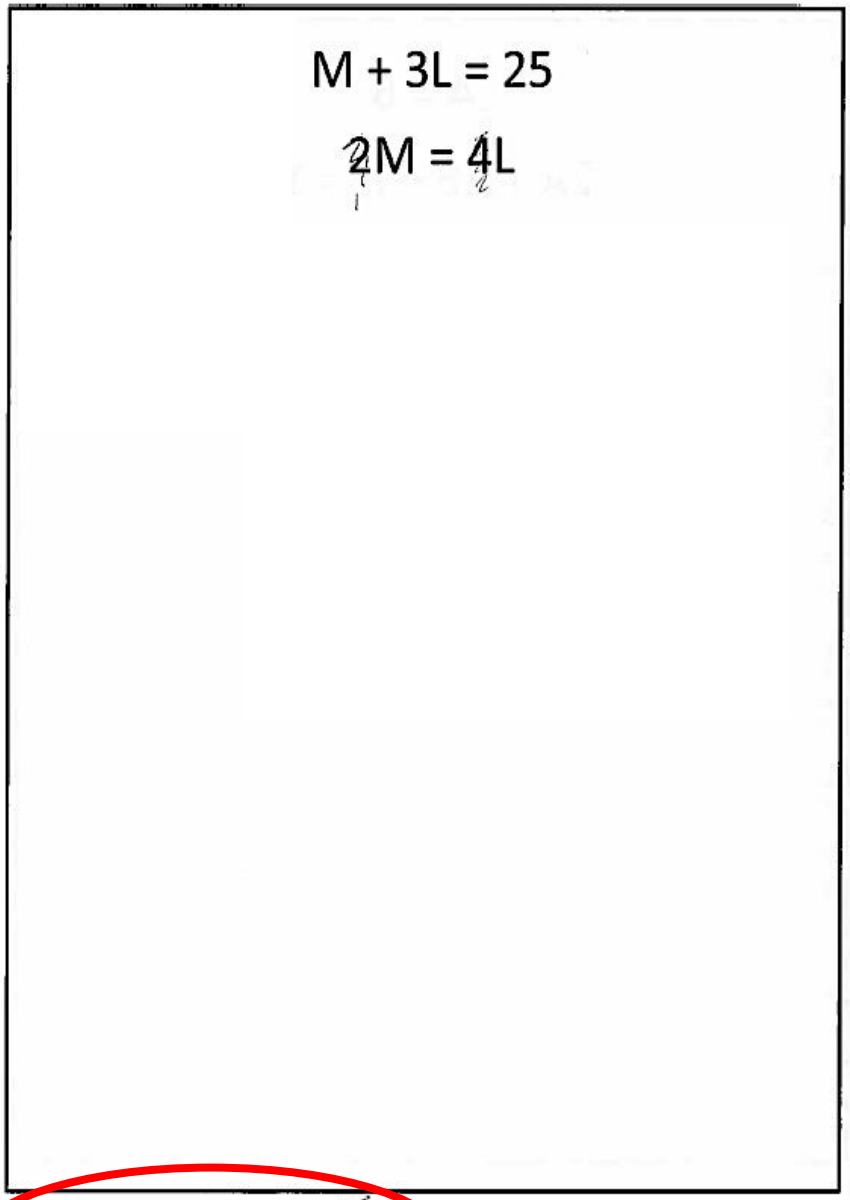
Test item of lesson 6
(the final lesson)



early algebra

the **bodily experience** with the hanging mobile becomes a **cognitive hook**

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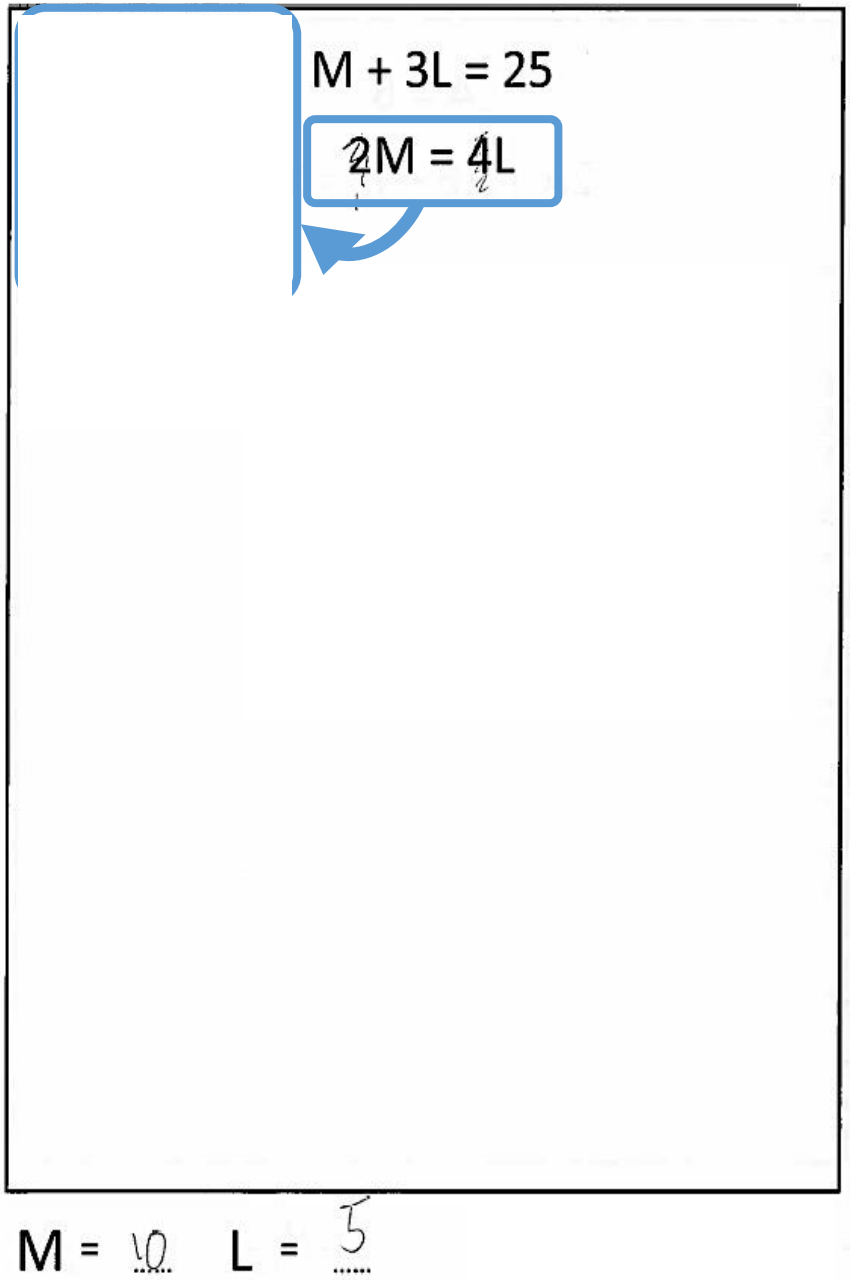

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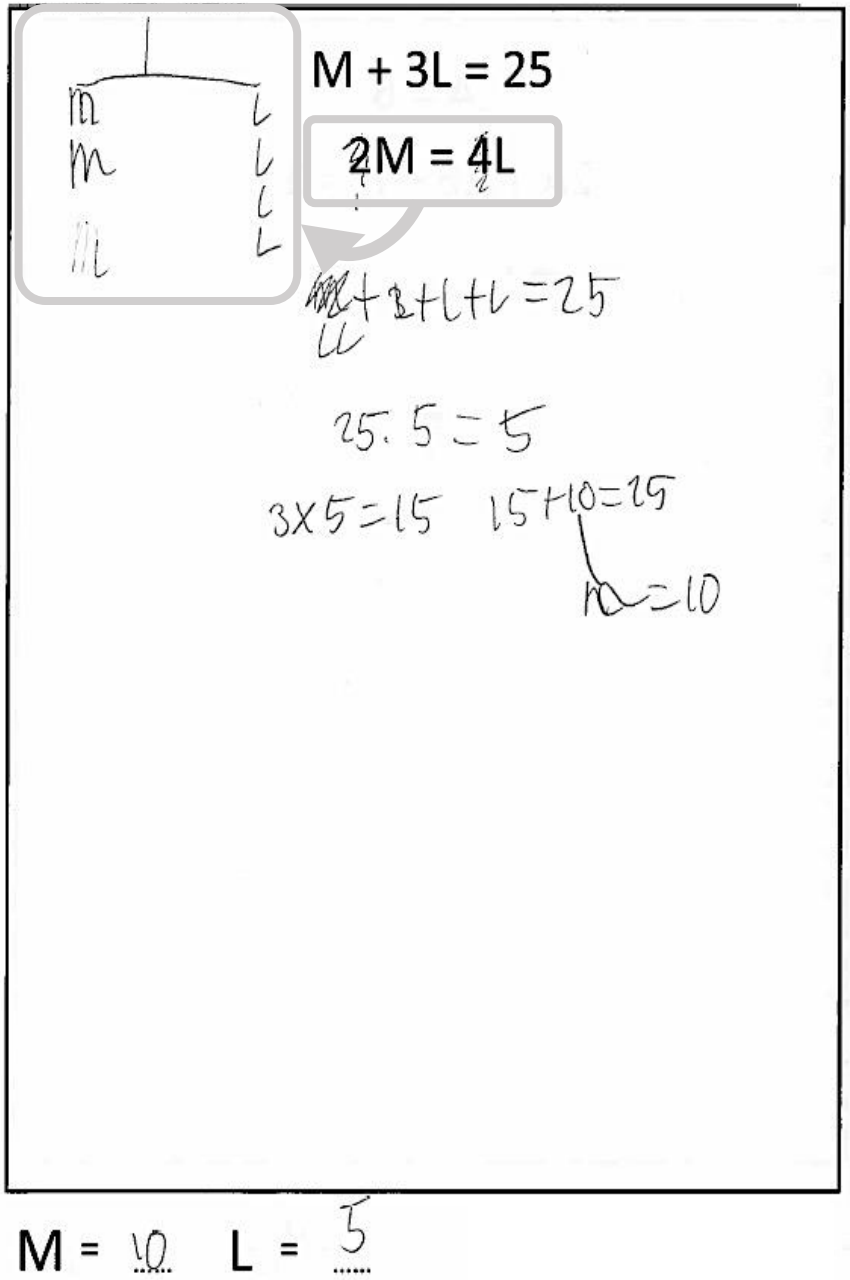

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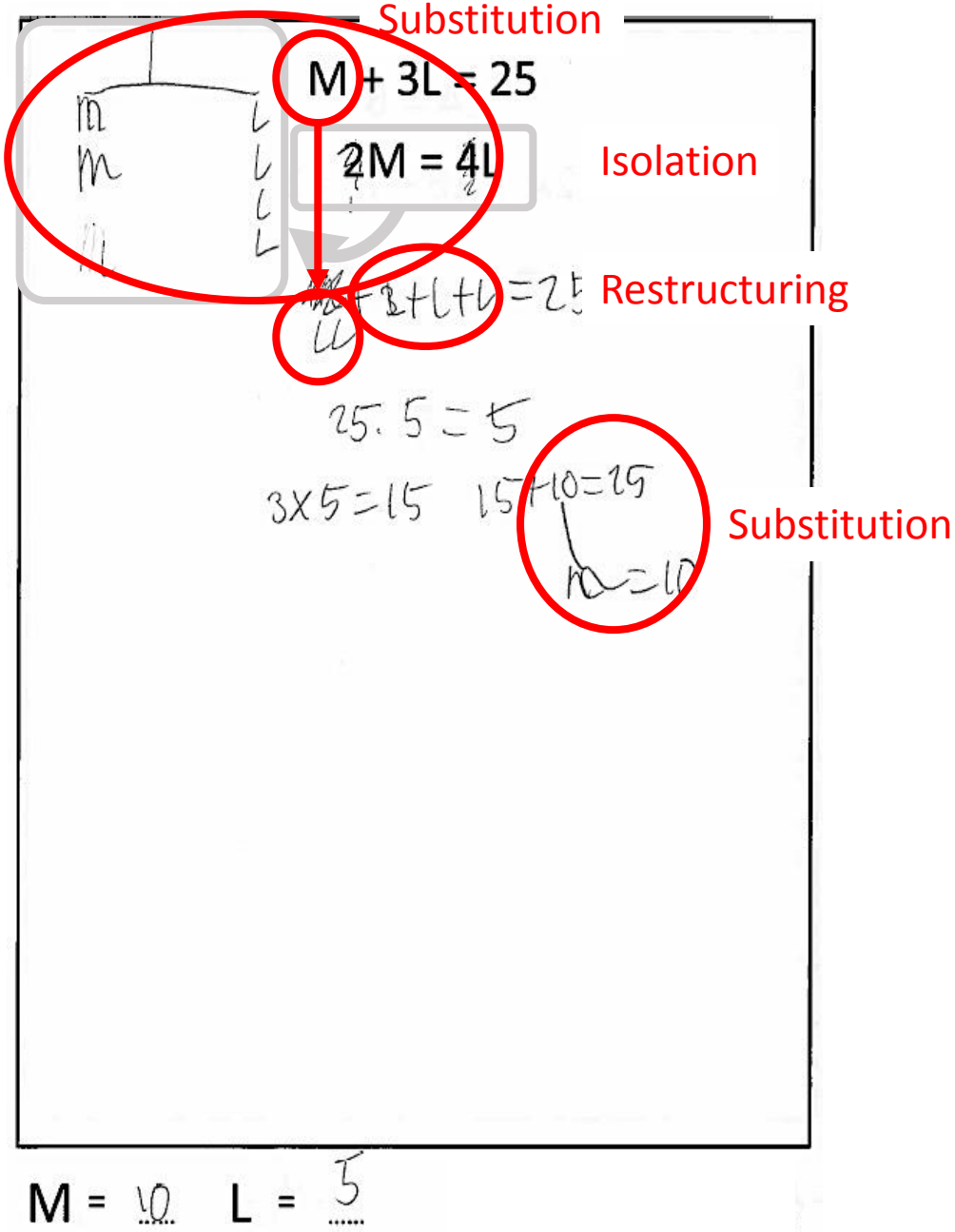

- The role of embodiment in developing mathematical higher-order thinking

dynamic data modeling

Background

In the Netherlands, graphical representations of dynamic situations are rare in the primary school curriculum

International research shows that young children are able to reason about graphical representations of dynamic data (e.g., DiSessa et al., 1991; Nemirovsky, Tierney & Wright, 1998)

Research question

How can primary school students' understanding of graphical representations of dynamic data be fostered?

dynamic data modeling

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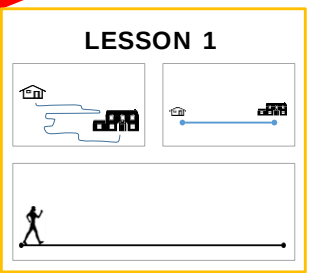

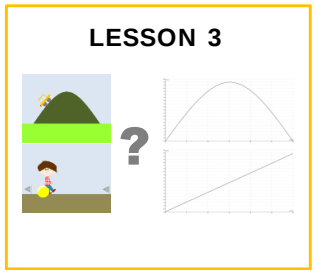

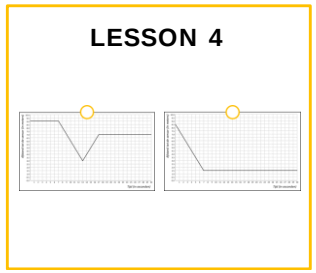

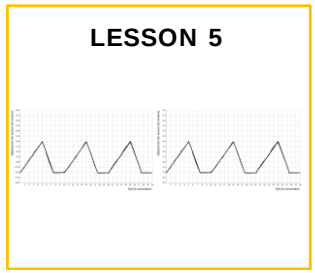

Teaching sequence of six lessons

Focus on: *Reasoning about graphical representations of change*

Specifically: *Reasoning about, and interpreting, time-distance-graphs*

Context: *Moving in front of a motion sensor*

Embodiment/grounding: *Experience of moving through space – graph (covariation)*

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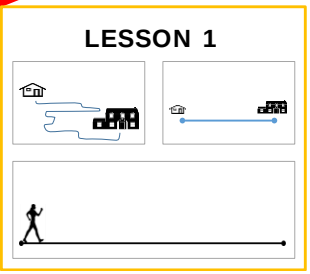
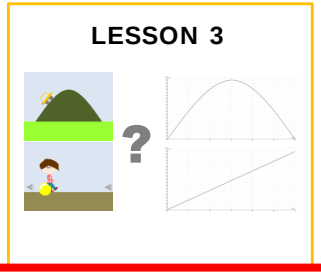
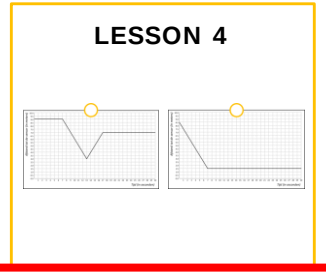
dynamic data modeling

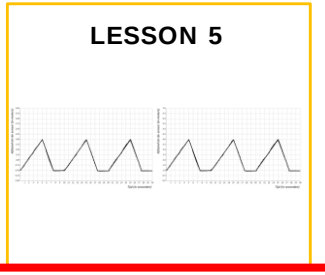
Research question

How can primary school students' understanding of graphical representations of dynamic data be fostered?









By using a motion sensor students can experience how their own movements relate to the graphical representation

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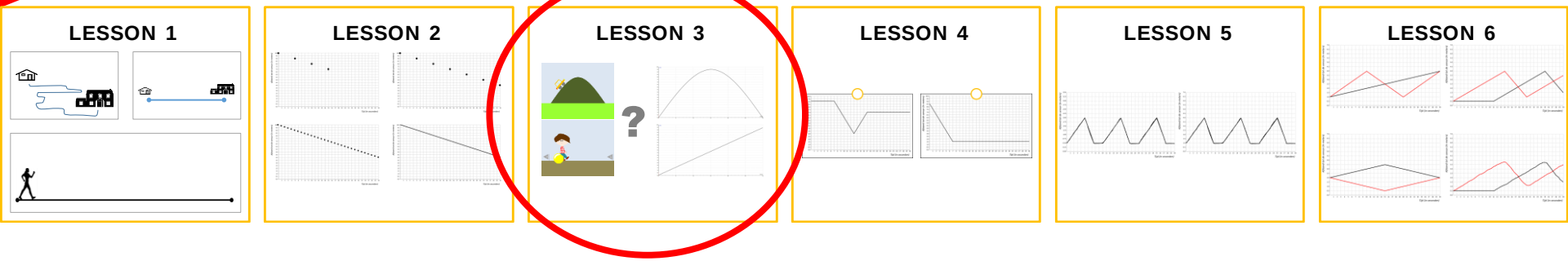

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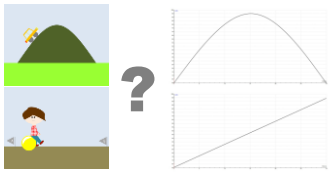
dynamic data modeling

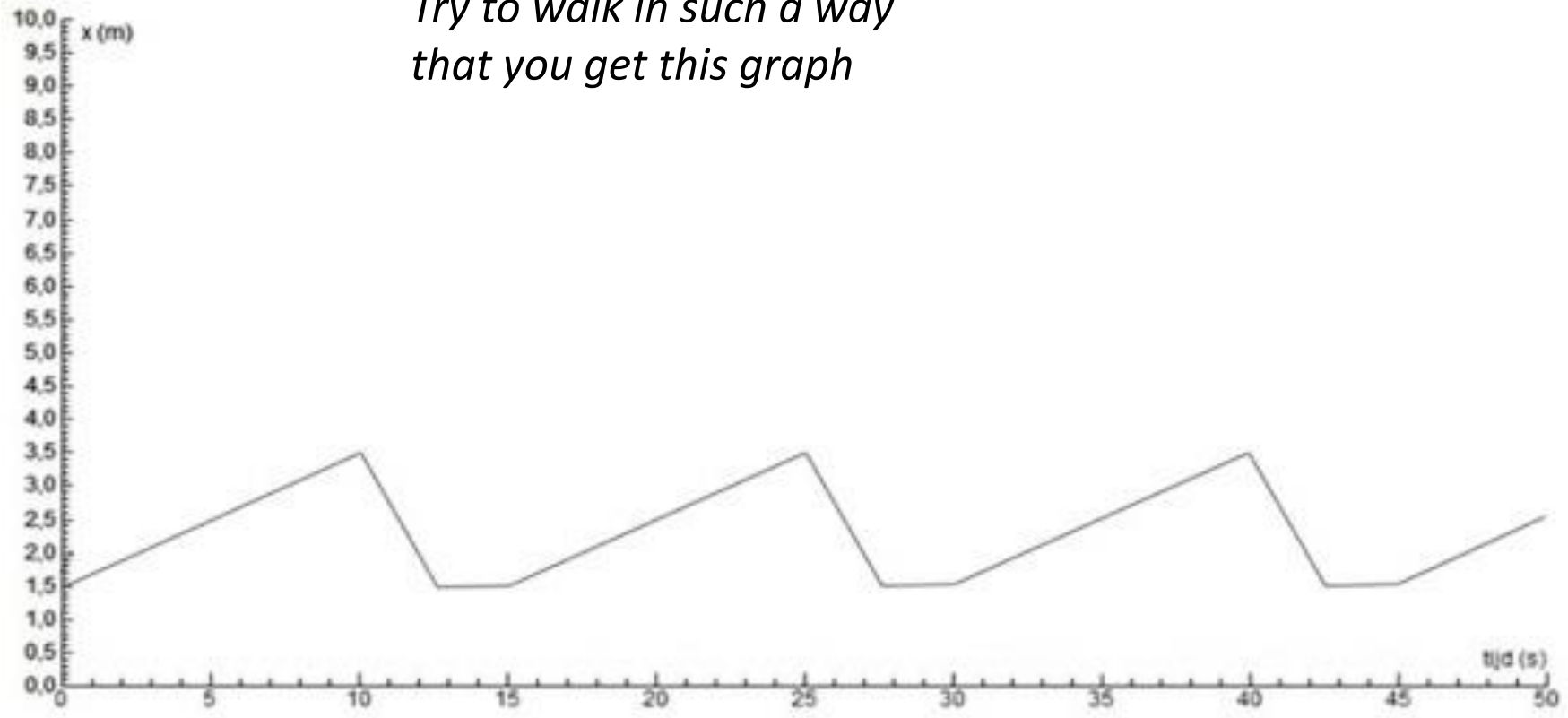
LESSON 3



CONTINUOUS GRAPHS OF 'DISTANCE TO'

Task

Try to walk in such a way that you get this graph



- The role of embodiment in developing mathematical higher-order thinking

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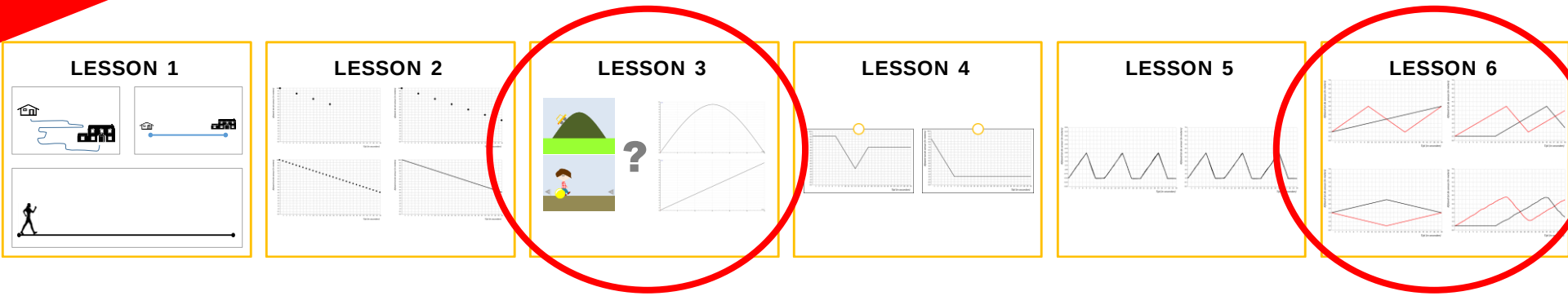

Teaching sequence of six lessons

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Specifically: *Reasoning about and interpreting time-distance-graphs*

Context: *Moving in front of a motion sensor*

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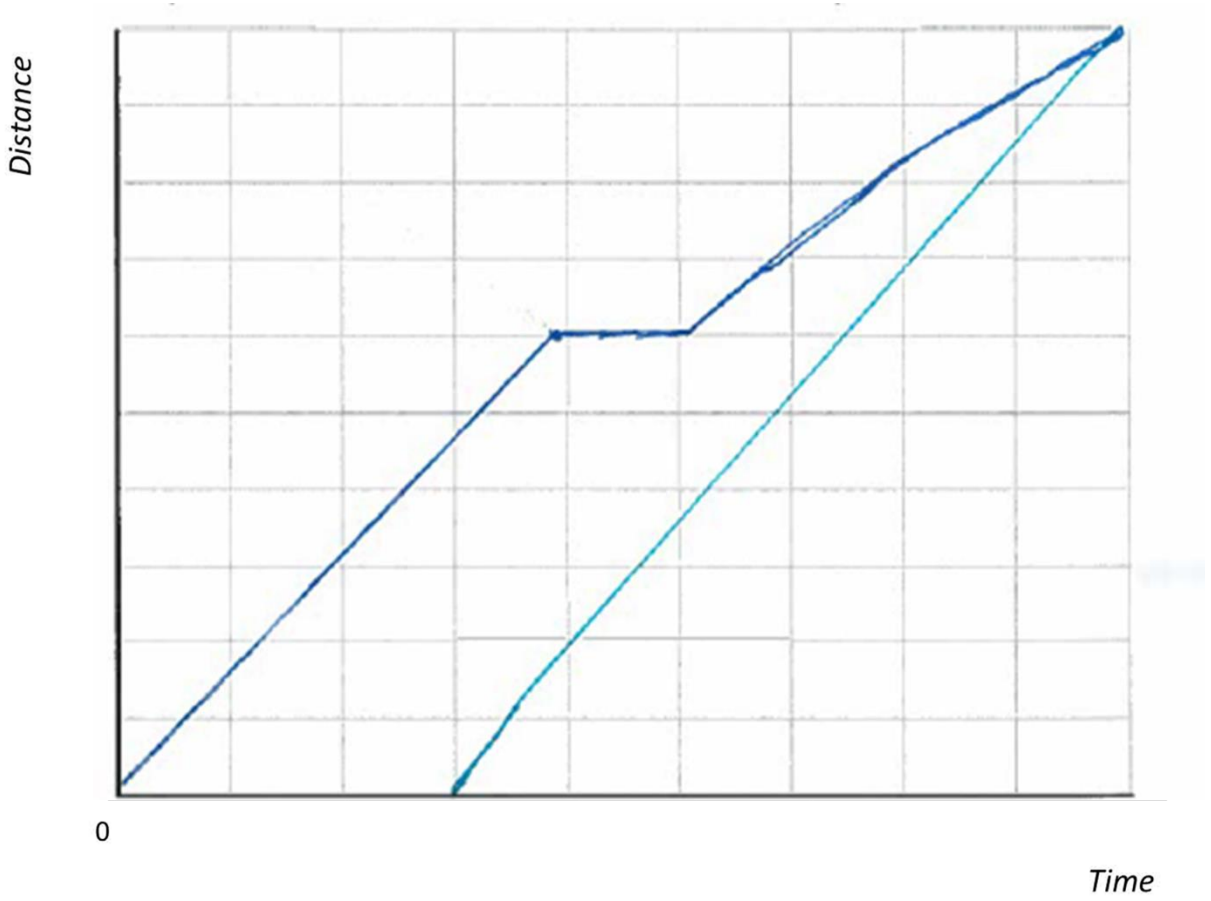
dynamic data modeling

From home to school

Lisa leaves home earlier than her brother Jan. Halfway Lisa waits for Jan. They arrive at school together.

Draw a graph that could fit this description.



Test item
Lesson 6

probability

Background

In the Netherlands, probability is first taught in secondary school

International research shows that young children are able to reason about probability (e.g., Bryant & Nunes, 2012)

Research question

How can primary school students' probabilistic reasoning be fostered?

probability

Research question

How can primary school students' probabilistic reasoning be fostered?

Teaching probability

Common approach

- Doing experiments
- Seeing what comes out
- Explaining the results

Our approach

- Exploring the **sample space**
- Predicting what comes out
- Doing one experiment
- Doing more experiments

probability

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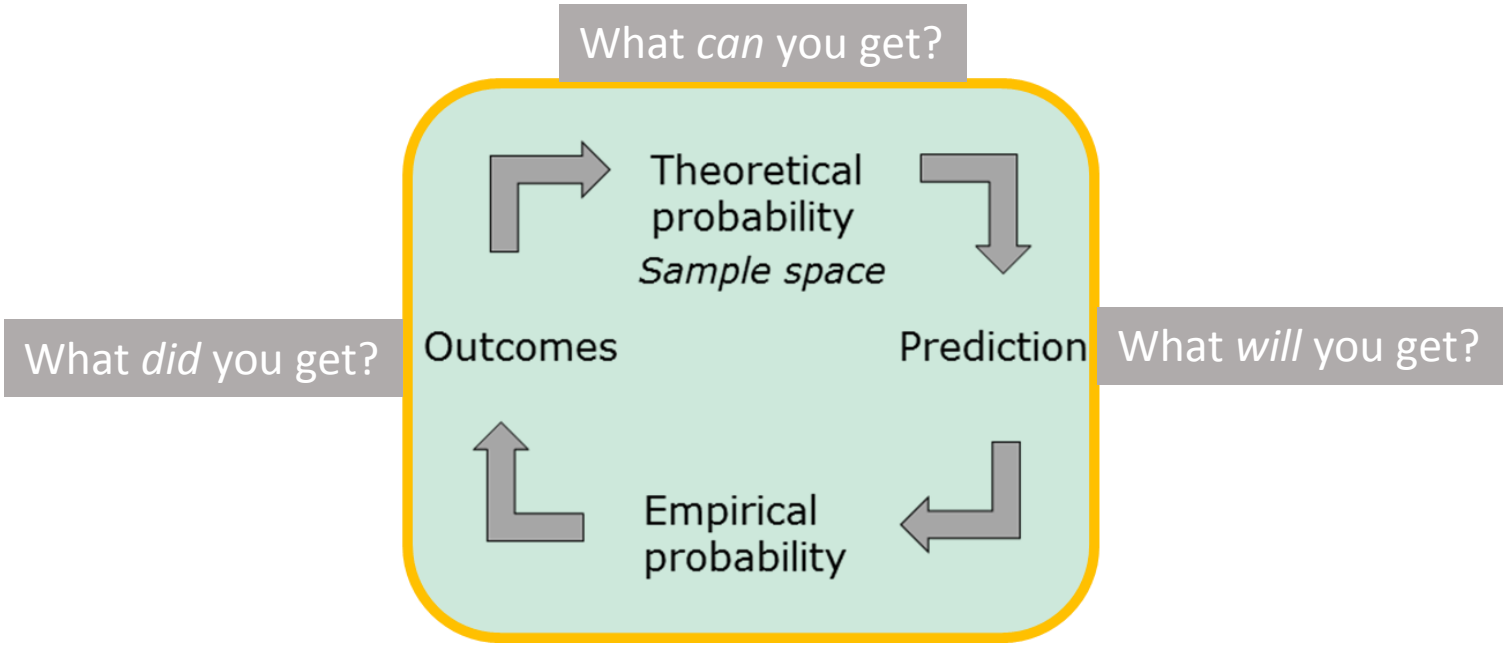

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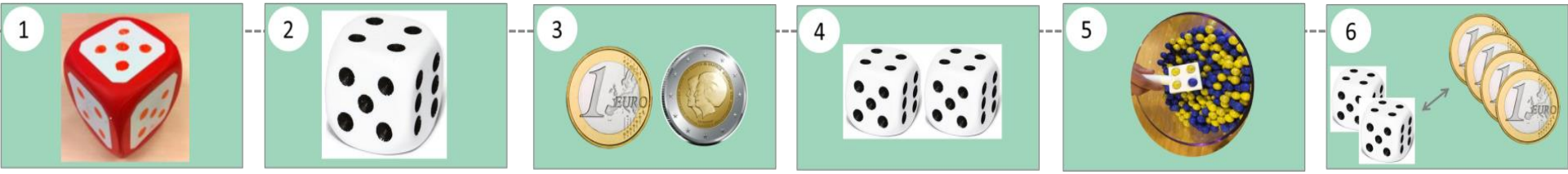

Teaching sequence of six lessons

Focus on: *Use sample space as a starting point for probabilistic reasoning*

Specifically: *Switch perspectives between:*

- *Unpredictability <-> Predictability*
- *Theoretical probability <-> Empirical probability*
- *Elementary results <-> Types of results*

Context: *Physical chance generators and simulations*

Embodiment/grounding:

+ Experience unpredictability of single outcome(s)

+ Experience the gradual construction of predictable distribution

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probability

Research question

How can primary school students' probabilistic reasoning be fostered?

What *can* you get?

Theoretical probability

Sample space

What *did* you get?

Outcomes

Prediction

What *will* you get?

Empirical probability

- The role of embodiment in developing mathematical higher-order thinking

probability

Task

1. What results can you get with two coins
2. Predict who will win..

HH or HT or TH or TT

Lisa

		
Tim chooses two times heads	*Lisa chooses heads and tails*	*Richard chooses two times tails*

Two coins are flipped 100 times.

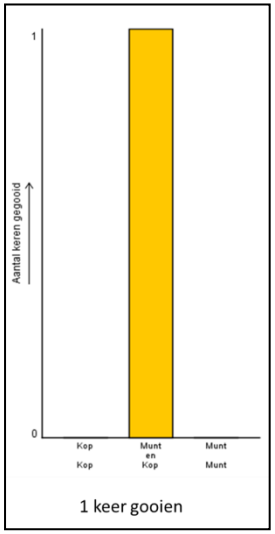
probability

3. Try it out and stack the pieces of wood (HH, HT, TT)

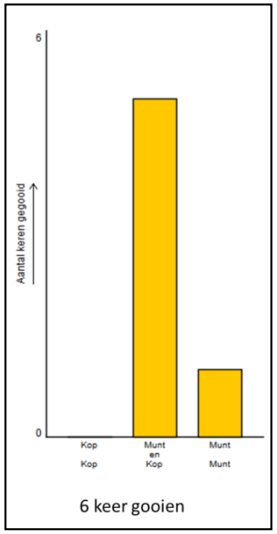
- The role of embodiment in developing mathematical higher-order thinking

probability

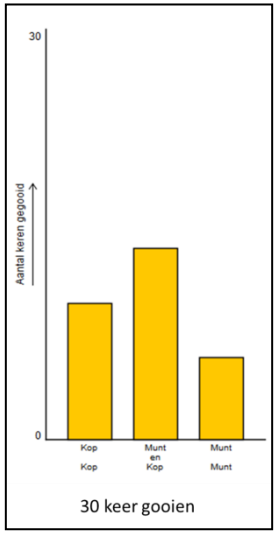
4. Simulate this on the computer



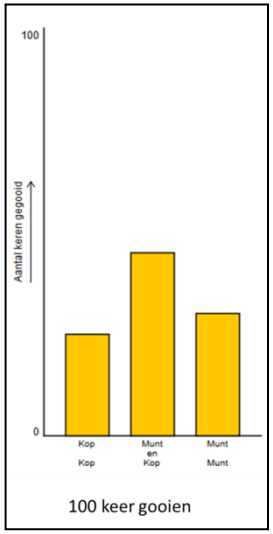
1 keer gooien



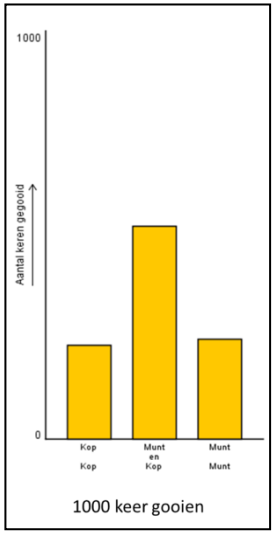
6 keer gooien



30 keer gooien



100 keer gooien



1000 keer gooien

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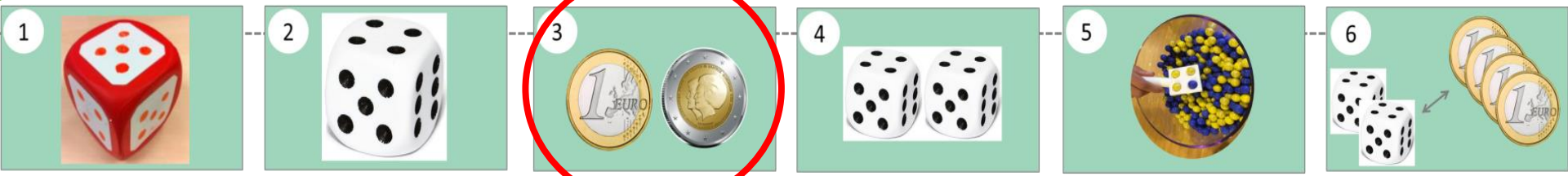

What *can* you get?

1. What results can you get with two coins

Theoretical probability

Sample space

What *will* you get?

2. Predict who will win..

Prediction

Outcomes

What *did* you get?

4. Simulate this 1000x on the computer

Empirical probability

3. Try it out and stack the pieces of wood

probability

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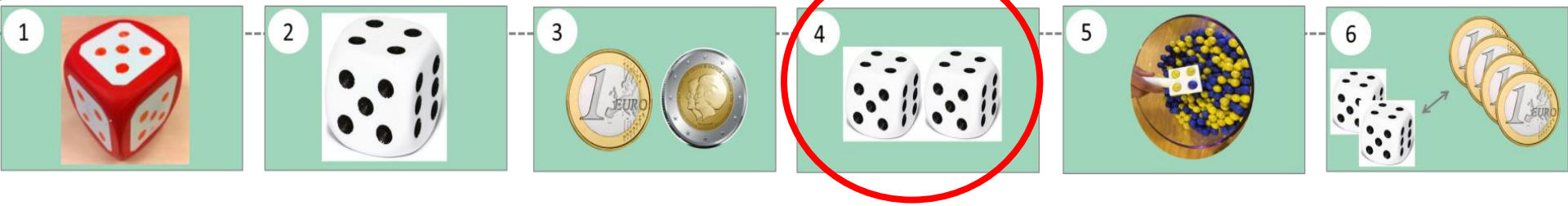


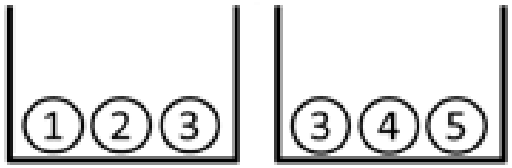
- The role of embodiment in developing mathematical higher-order thinking

probability

Test item
Lesson 4

From every bucket a ball is drawn, without looking. The numbers on the balls are added.

- *Daan wins if the total is '4'*
- *Emma wins if the total is '5'*
- *Sem wins if the total is '6'*



Who is most likely to win?

How do you know?

Test item
Lesson 4

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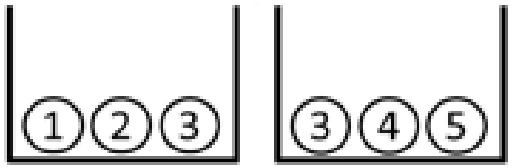
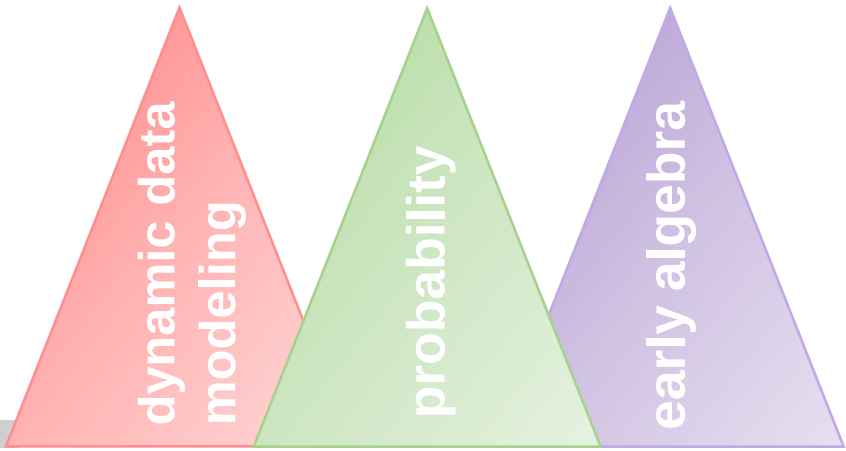


Who is most likely to win? Sem

How do you know?

6 kan op 3 manieren gepakt worden
2 4 | 1 5 | 3 3
5 op 2 manieren
en 4 op een manier dus heeft Sem de meeste kans om te winnen.

You can get 6 in 3 different ways:
2 4 | 1 5 | 3 3
5 in two ways 2 3 | 1 4
and 4 in one way 1 3
so Sem has the most chance to win



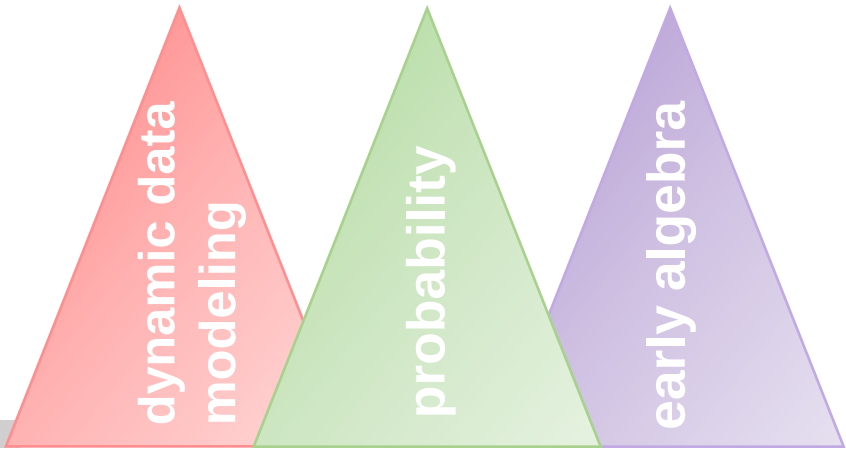
Conclusion for today

Using different conceptualizations of embodiment

Students' algebraic reasoning (namely, understanding of equality and use of algebraic strategies) **can be elicited**

Students' understanding of relations between movement and its graphical representation (in a time-distance graph) **can be elicited**

Students' understanding of the relation between sample space and probability (of an event) **can be elicited**



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