The role of embodiment in developing higher-order mathematical thinking in primary education

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Interview with fashion designer Iris van Herpen
in de Volkskrant, October 28, 2017

The form comes into existence during the process of folding, while her head and hands are working together and at the same time: “A part of the design process is unconscious, I literally let my hands do the work. Because I have been doing this for such a long time, knowledge and intuition went into my hands that go further than what I can consciously think of.” The body saves, according to her, like in dancers and musicians, knowledge through experience and repetition. “This leads me to quickly understand the material and know what I can do with it. It is completely in my hands.”
The role of embodiment in developing higher-order mathematical thinking in primary education
Beyond Flatland
in primary school mathematics education

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• The role of embodiment in developing mathematical higher-order thinking
Beyond Flatland in primary school mathematics education

Develop a teaching sequence of six lessons

Learning facilitators:
- Embodied cognition
- Representational redescription theory

Sequencing of tasks:
- Variation theory

Data:
- Videos of students working in the lessons
- Students’ written work in the lessons
- Students’ responses on lesson-specific test items

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Background

In the Netherlands, teaching algebra starts in the first grades of secondary school (12-13 years)

International research provides evidence that (early) algebra can be taught in primary school (e.g., Brizuela & Schliemann, 2004; Kaput et al., 2008)

Research question

How can primary school students’ algebraic reasoning be fostered?
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Teaching sequence of six lessons

Focus on: *Algebraic reasoning with linear equations*

More specifically: *Reasoning with, and about, unknowns using algebraic strategies*

Context: *Working with a hanging mobile*

Embodiment/grounding: *Experience of balance - equivalence*

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Research question:
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Teaching sequence of six lessons:

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How can primary school students' algebraic reasoning be fostered?

Teaching sequence of six lessons

Context → Model

Focus on: Algebraic reasoning with linear equations

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Context → Model

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What can you do to keep the hanging mobile straight?

- Reactivating the concept of equivalence
- Eliciting algebraic strategies
  - Restructuring
  - Changing sides
  - Changing order of bags on the same side
  - Isolation
  - Taking away similar bags on both sides
  - Taking away different bags on both sides
- Substitution
  - Replacing bags by bags of another color

early algebra
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Embodyment/grounding: Experience of balance - equivalence

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Test item of lesson 6 (the final lesson)

M + 3L = 25
2M = 4L

M = 10, L = 5

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The role of embodiment in developing mathematical higher-order thinking

Early algebra

The bodily experience with the hanging mobile becomes a cognitive hook

Test item of lesson 6 (the final lesson)

\[ M + 3L = 25 \]
\[ 2M = 4L \]

\[ M = \_\_\_ \quad L = \_\_\_ \]
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Early algebra

The bodily experience with the hanging mobile becomes a cognitive hook.

Test item of lesson 6 (the final lesson)

- The role of embodiment in developing mathematical higher-order thinking
Test item of lesson 6 (the final lesson)

The bodily experience with the hanging mobile becomes a cognitive hook.

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- Early algebra

- Substitution
- Isolation
- Restructuring
- Substitution

M + 3L = 25
2M = 4L

25 \times 5 = 5
3 \times 5 = 15
15 + 10 = 25
M = 10

L = \ldots
Background

In the Netherlands, graphical representations of dynamic situations are rare in the primary school curriculum.

International research shows that young children are able to reason about graphical representations of dynamic data (e.g., DiSessa et al., 1991; Nemirovsky, Tierney & Wright, 1998).

Research question

How can primary school students’ understanding of graphical representations of dynamic data be fostered?
Research question

How can primary school students’ understanding of graphical representations of dynamic data be fostered?

Teaching sequence of six lessons

Focus on: *Reasoning about graphical representations of change*

Specifically: *Reasoning about, and interpreting, time-distance-graphs*

Context: *Moving in front of a motion sensor*

Embodiment/grounding: *Experience of moving through space – graph (covariation)*

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By using a motion sensor students can experience how their own movements relate to the graphical representation.

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Research question

How can primary school students’ understanding of graphical representations of dynamic data be fostered?

Teaching sequence of six lessons

Focus on:  *Reasoning about graphical representations of change*

Specifically:  *Reasoning about, and interpreting, time-distance-graphs*

Context:  *Moving in front of a motion sensor*

Embodiment/grounding:  *Experience of moving through space – graph (covariation)*

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Task

Try to walk in such a way that you get this graph

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Research question

How can primary school students’ understanding of graphical representations of dynamic data be fostered?

Teaching sequence of six lessons

Focus on: Reasoning about graphical representations of change

Specifically: Reasoning about and interpreting time-distance-graphs

Context: Moving in front of a motion sensor

Embodiment/grounding: Experience of moving through space - graph

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From home to school
Lisa leaves home earlier than her brother Jan. Halfway Lisa waits for Jan. They arrive at school together.

Draw a graph that could fit this description.
Background

In the Netherlands, probability is first taught in secondary school.

International research shows that young children are able to reason about probability (e.g., Bryant & Nunes, 2012).

Research question

How can primary school students’ probabilistic reasoning be fostered?

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Teaching probability

**Common approach**
- Doing experiments
- Seeing what comes out
- Explaining the results

**Our approach**
- Exploring the sample space
- Predicting what comes out
- Doing one experiment
- Doing more experiments

**Research question**
How can primary school students’ probabilistic reasoning be fostered?
Research question
How can primary school students’ probabilistic reasoning be fostered?

What can you get?

What did you get?

What will you get?

Empirical probability

Predictions

Outcomes

Theoretical probability

Sample space

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Research question

How can primary school students’ probabilistic reasoning be fostered?

Teaching sequence of six lessons

Focus on: Use sample space as a starting point for probabilistic reasoning

Specifically: Switch perspectives between:
  • Unpredictability <-> Predictability
  • Theoretical probability <-> Empirical probability
  • Elementary results <-> Types of results

Context: Physical chance generators and simulations

Embodiment/grounding:
  + Experience unpredictability of single outcome(s)
  + Experience the gradual construction of predictable distribution

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Research question

How can primary school students’ probabilistic reasoning be fostered?

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Task
1. What results can you get with two coins
2. Predict who will win
3. Try it out and stack the pieces of wood (HH, HT, TT)
4. Simulate this 1000x on the computer

Lisa

HH or HT or TH or TT

<table>
<thead>
<tr>
<th>Tim chooses two times heads</th>
<th>Lisa chooses heads and tails</th>
<th>Richard chooses two times tails</th>
</tr>
</thead>
</table>

Two coins are flipped 100 times.
3. Try it out and stack the pieces of wood (HH, HT, TT)
4. Simulate this on the computer

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Research question

How can primary school students’ probabilistic reasoning be fostered?

1. What results can you get with two coins

2. Predict who will win..

3. Try it out and stack the pieces of wood

What can you get?

4. Simulate this 1000x on the computer

What did you get?

What will you get?

Outcomes

Theoretical probability

Sample space

Prediction

Empirical probability

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Research question

How can primary school students’ probabilistic reasoning be fostered?
From every bucket a ball is drawn, without looking. The numbers on the balls are added.

- Daan wins if the total is ‘4’
- Emma wins if the total is ‘5’
- Sem wins if the total is ‘6’

Who is most likely to win?

How do you know?
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Test item
Lesson 4

From every bucket a ball is drawn, without looking. The numbers on the balls are added.

- Daan wins if the total is ‘4’
- Emma wins if the total is ‘5’
- Sem wins if the total is ‘6’

Who is most likely to win? Sem

How do you know?

You can get 6 in 3 different ways:
2 4 | 1 5 | 3 3
5 in two ways 2 3 | 1 4
and 4 in one way 1 3
so Sem has the most chance to win
Conclusion for today

*Using different conceptualizations of embodiment*

**Students’ algebraic reasoning** (namely, understanding of equality and use of algebraic strategies) *can be elicited*

**Students’ understanding of relations between movement and its graphical representation** (in a time-distance graph) *can be elicited*

**Students’ understanding of the relation between sample space and probability** (of an event) *can be elicited*
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