Embodied mathematical learning using touchscreen tablets: an Eye tracking study

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Sample screen shots from enacting the proportion app. In all conditions the pre-set proportion is 1:2 and the color feedback will be green only when the left finger’s position is half the right finger’s position. The top section (a, b, c, d) only involves proportional relation. In the bottom section (e, f, g, h) we are introducing geometrical concepts such as angle, slope, scaling and Cartesian point in relation with proportional change.
Whether and how the visual information and sensorimotor schemes are related to the students’ emerging proportional reasoning
Design

Three studies:
- Fifth and sixth Grade (mean age = 11 yrs & 3 m)
- VMBO schools (mean age = 13 yrs & 5 m)

Two main conditions
- Parallel
- Orthogonal

Data gathered included
- action-logging (hand movement logs)
- eye-tracking
- videography
Set up
Proportion as a Relation

• Discover the hands as two main variants
• Realize the two variants are related
  • There is a relation
• How are they related? What is that relation?
  • It is about their relative height/length compared to each other.
    • Is it an “equal” relation?
    • Is it a fixed difference?
• How do different designs elicit this knowledge
First Study- (Parallel)

Explore phase
Bimanual Coordination skill
  Learned the ‘action’ and invented their own strategies
Grid
  “Rule” Discovery
Grid & Number
  Verification
Thinking aloud all through
Explore phase

- Systematic pattern of testing strategies to detect variants
  - Move hands at the same height
  - Move one of hand only
  - Move one of them faster
  - Move hands in opposite direction
- Once they have found a couple of greens
  - Sometimes they try the corresponding inverse relation
- At the end they knew the two hands are related
- They don’t know the exact relation yet.
Bimanual coordination skill

- Increasingly emerging momentums of intertwined perception-action-reasoning coupling
  - When movement is explorative gaze is aligned with it, switching and following the leading finger and at times measuring the height.
  - They expand their repertoire faster
  - Changes happen more quickly in time
  - They have a pool of possible behavior, and among it they select the right one

- Emergence of Attentional Anchors
Attentional Anchors

• Emerges when a subject with certain abilities interacts with the environment.
• Helps controlling the activity.
• Subject/environment elements, features or aspects.
  • Environmental or task constraint
  • Self-discovered or proposed
  • Implicit or explicit
  • Temporary or permanent
Juggling cascade

Elementary actions of tossing and catching with right hand, left hand, looking.
Concatenation of tosses and catches while looking.
A tall rectangle or the area around the zeniths can work as AA.

Tall rectangle as AA
Area around the zeniths as AA
In Parallel condition gaze pattern seems relevant to the conceptual knowledge behind proportional reasoning.
Elementary actions moving left hand, right hand, looking. Coordination of left, right hand movements and looking to make a green. Keeping the bars green all the time.

The learner discovers a triangle as AA to coordinate the actions in order to keep the bars green all the time.
Various Observed Patterns

Parallel condition with bars

Quantified Analysis proved these were most frequent patterns
Relational behavior

• Evident from Conversation & Action & gaze segments.
  • More and more instances that was a result of comparison and analogy making to extract pieces of relational knowledge
    “One has to be always higher than the other ”
    “Right one has to be higher than left one”

  While their gaze was shifting more and more towards the triangle of focus

• Continuing this behavior promoted acting and thinking more relationally
Solution strategies

- Emerge from a cognitive conflict of focusing on fixed vertical interval, to a proportionally changing interval

- Ordered and progressive, with students progressing from mostly additive (C and D) to mostly multiplicative strategies (E and F)

*Note on abbreviations: LC = left-hand cursor; RC = right-hand cursor; Δ = magnitude of interval between cursors (vertical and diagonal variants); v = velocity*
Second-Study Orthogonal

• What if we turn it into a two dimensional space
  • Same principle behind the movement
    • Coordination of two fingers at different speed
  • Same visual feedback
Highlights from Orthogonal

(shorter) Exploration and less rich
(faster grasp on) bimanual coordination
(more Diverse) attentional anchors
Diverse reasoning and conceptualization

End result was the same once they used the grid and numbers to find the relation
Explore less, Find green fast

• Dimension of the working space (ipad) helped them finding greens faster and easier

  • **BUT** stopped them from realizing (naturally) that there is a relation between the position of their two hands

  • **For some** it hindered the whole learning experience by NOT eliciting an Explore and Test and Fail and Connect and Relate experience in a coherent and helpful way.
Diverse Attentional Anchors

- AAs that were not necessarily relevant to the proportional change
- Not always coupled with immediate past-future acting and reasoning behavior
Talk differently?

• Initially the hands seemed like two separate entities
  • Even though they were good at coordinating them, the didn’t talk about any relevant relation between them
• (Pushed to) see they are somewhat related
  “They are closer at the beginning and further apart at the end”
• How much apart?
  “The left one is closer to the middle (of the Y axis) than right one is to the middle (of X axis)”

• Their comparison didn’t share the same base
• Their relational thinking didn’t develop similarly
That they, let's say, that the angle has to be on the same line.
When he sees the line we see dots

Experimenter/Design is not ready for the right follow up interaction

Grids are introduced and he is lost
Third Study

Master Thesis, Loes Boven, 2017

- An interaction protocol more open to ideas
  - No imposing questions (or at the wrong time)
  - Be prepared for different path or reasoning

- Allow for more exploration time

- Younger group of children
  - Who talked more, freely and more eager to discover
Include (Geometrical) Cue

Explicitly hint that the two hand positions are related and relevant

How do the solution strategies and/or gaze patterns emerge differently?
• In general similar states of progression occurred

• Expanded domain of thinking aloud and reasoning
  
  • Length
  
  • Line / angle
  
  • Shape

• The differences were more interesting
Length

• At Initial states the connection between the two length is lost

  “Left must be below the top, Right near the right side of the screen”
  “They are always around the same place, both below or both in the middle”
  “They are both in the start, both in the middle or both at the end”
  “They are both further apart, but right is further”

• With the help of grid and numbers as well as hints from instructor they progress to additive and multiplicative statements.
They either imagine a line or asked about one (at the end)

- My fingers must be diagonally opposite to each other
- My fingers point to each other
- My fingers stay on one line
- Every time it gets green, a new line emerges
- It is a steep line. The steepness does not change when I move, but the line does get bigger
- **The steepness does change a bit every time!**
Shape

• It is exactly the same shape, but bigger and smaller
• I first thought it had to be the same length and width, but that was not the case because it is a rectangle

• It is a rectangle that is two squares
Eventually

- Ay and Ax have the **same** proportional relation
- Diagonal lines (C) have **the same** relation (same slope)
- The shape is always proportionally **the same**
What is missing?

• What we thought would work as a cue, worked as the most hindering way
  • In the rectangle condition all they do is focusing on the shape as a whole without being able to break it to its Component
  • The relation between “the length” of position of the two hands is postponed even later

• The connection
  • They are all interconnected and about the same proportional change
  • Not much geometrical or knowledge exactly
It is all about the design and Interaction
Parallel

• The interval is more salient and after a while the only thing they attend to
• The **importance** and **relevance** of height/length is salient
• The fact that there is a relation between the two is constantly visible and promoted.
Orthogonal

• The equivalent to the interval is less salient
  • requires a spatial rotation to compare both lengths
• It is more difficult to determine the size of the interval
  • More difficult to track if the interval increases or decreased when moving.
• There is a whole lot more that they attend to
  • Evident from their talking and looking patterns
Different initial perspective

Parallel
• Side by Side comparison
  • One is more/higher than the other
  • How much more?
  • Fixed?
  • Changing?

Orthogonal
• Two separate entities
• Somewhat related
  • They are closer at the beginning and further apart at the end
  • How much apart?
  • Well one is closer to the middle than the other
Dynamical System View

Thelen & Smith 2006

- Dynamic patterns can be fleeting or very stable
  - Patterns are assembled for task specific purposes
  - The form and stability of patterns depends both on the immediate and more distant *history* of the system

- **Coupling** and **Continuity**
  - All the components of the developing system are continually linked and mutually interactive in the individual and between the individual and the environment
Dynamical System View

Thelen & Smith 2006

• Continuity
  • Each event in the body has
    • History
    • Here and now
    • Effect on the future

• Cumulative
  • Mental and physical activities are assembled in the moment and always as a function of systems history
  • Actions done in this moment, in turn set the stage for behavior in the next second, (minute, week, year)
    • Timelockness is critical
Design & Interaction

- Parallel design creates a more coherent learning experience
- Current Orthogonal designs elicits a richer and more diverse learning experience
  - Does not facilitate all the open learning paths
- Perhaps we should start with parallel
- Next take them to a more guided orthogonal experience
Lesson From Development

• All children learn to crawl, and walk and talk, etc.
  • It is still important to study them all.

• Here also learners eventually learn the task
  • It is important to study the process in micro-level

• Micro-level investigation of the mechanism can identify the hindering/facilitating components that are important in designing embodied interactive learning tools
Thank You