Embodied Design:
Developing a Research Program

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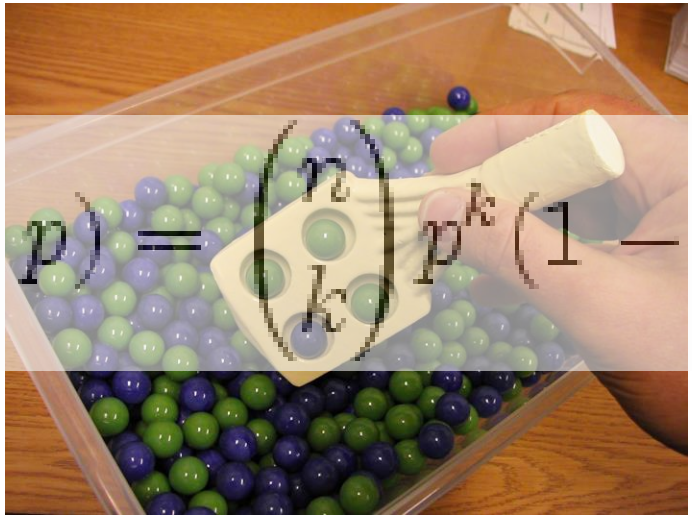
"Embodied Design in Interaction"
Universiteit Utrecht
October 30, 2017

$$f(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$



?

I can't make sense of this!

This makes no sense to me!

$$f(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$f(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

What is this?

What in the world is this?

What does this mean?

Why do you believe this?

What is the use of all these symbols?

$$f(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

Where do we begin?

What is the use of all these symbols? Why not begin by showing children the real thing so that they may at least know what you're talking about?

$$, p) = \binom{n}{k} p^k (1-p)^{n-k} {}^{n-k}$$

As a general rule—never substitute the symbol for the thing signified, unless it's impossible to show the thing itself; because the children's attention is so taken up with the symbol that they will forget what it signifies.

How do we engage children in interesting activities that will give rise to meaningful learning of big ideas they should understand and use, if they are to flourish in our wonderful, complicated world and make it a better place for themselves as well as all of us?

"The most usable meanings are those that are richly connected with **imagery** **action** and that tie into other meanings."

Pat Thompson, 2013

"What is the use of all these symbols? Why not begin by showing children the real thing so that they may at least know what you're talking about?

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Jean-Jacques Rousseau

Emile, Book III, 1762

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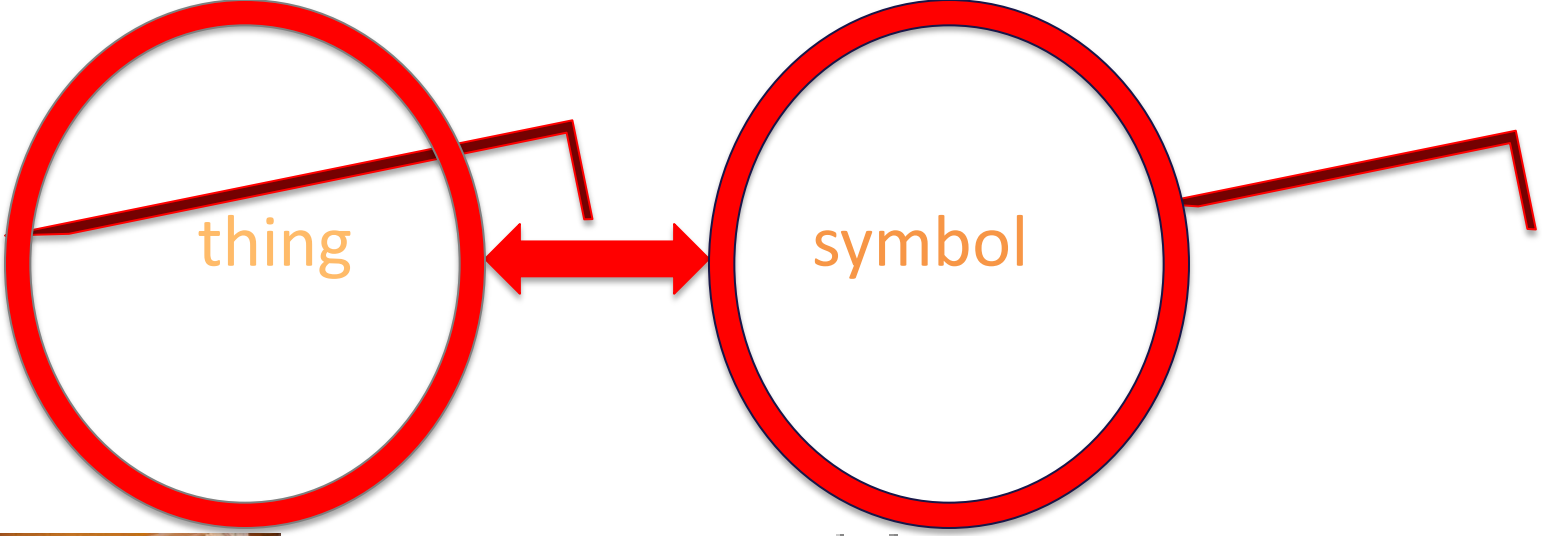


$$f(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

Embodied Design

Abrahamson, D. (2009). Embodied design: Constructing means for constructing meaning. *Educational Studies in Mathematics, 70*(1), 27-47.

Abrahamson, D. (2014). Building educational activities for understanding: An elaboration on the embodied-design framework and its epistemic grounds. *International Journal of Child-Computer Interaction, 2*(1), 1-16.

Abrahamson, D. (2009). Orchestrating semiotic leaps from tacit to cultural quantitative reasoning—the case of anticipating experimental outcomes of a quasi-binomial random generator. *Cognition and Instruction, 27*(3), 175-224.

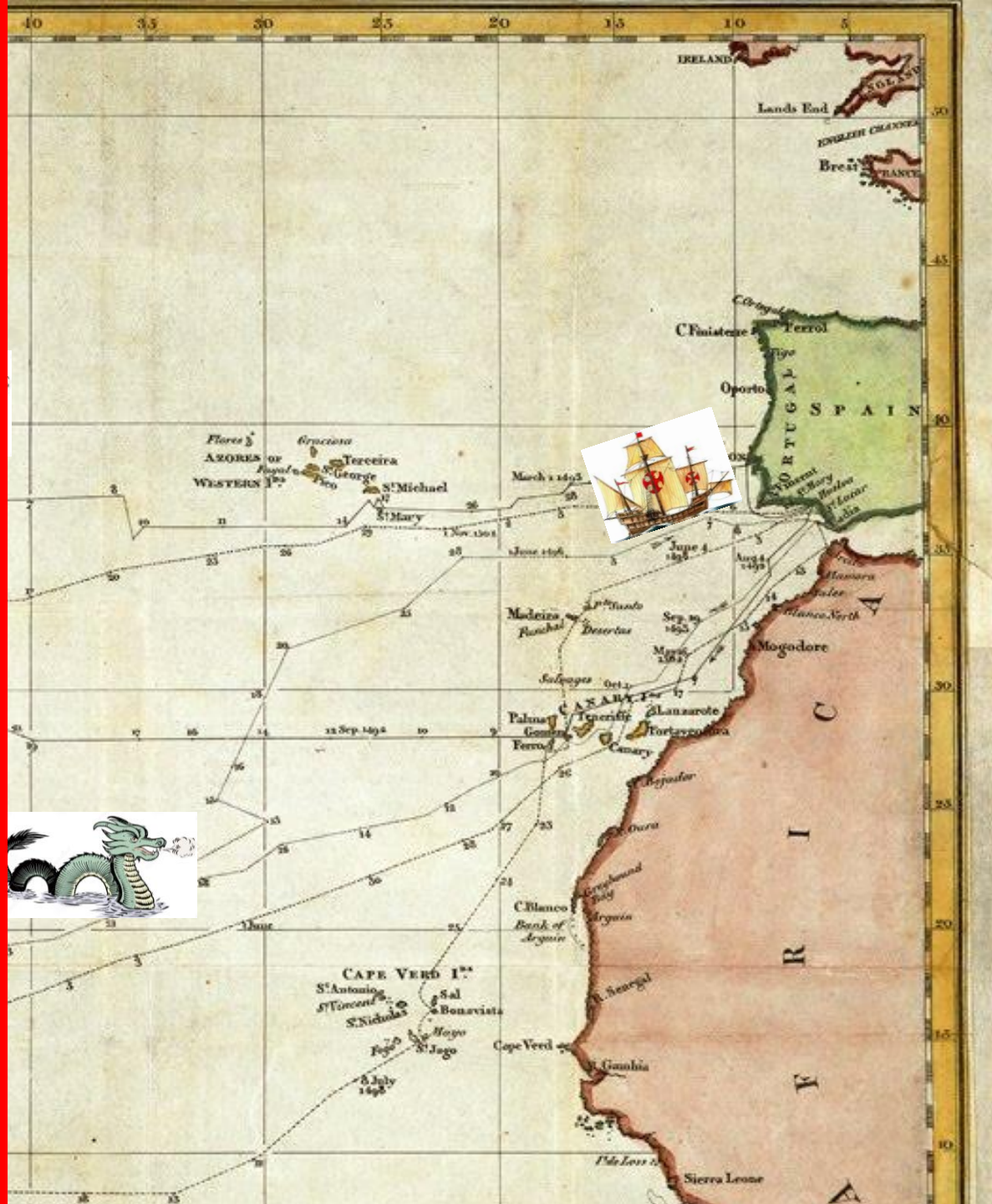
Abrahamson, D., & Sánchez-García, R. (2016). Learning is moving in new ways: The ecological dynamics of mathematics education. *Journal of the Learning Sciences, 25*(2), 203-239.

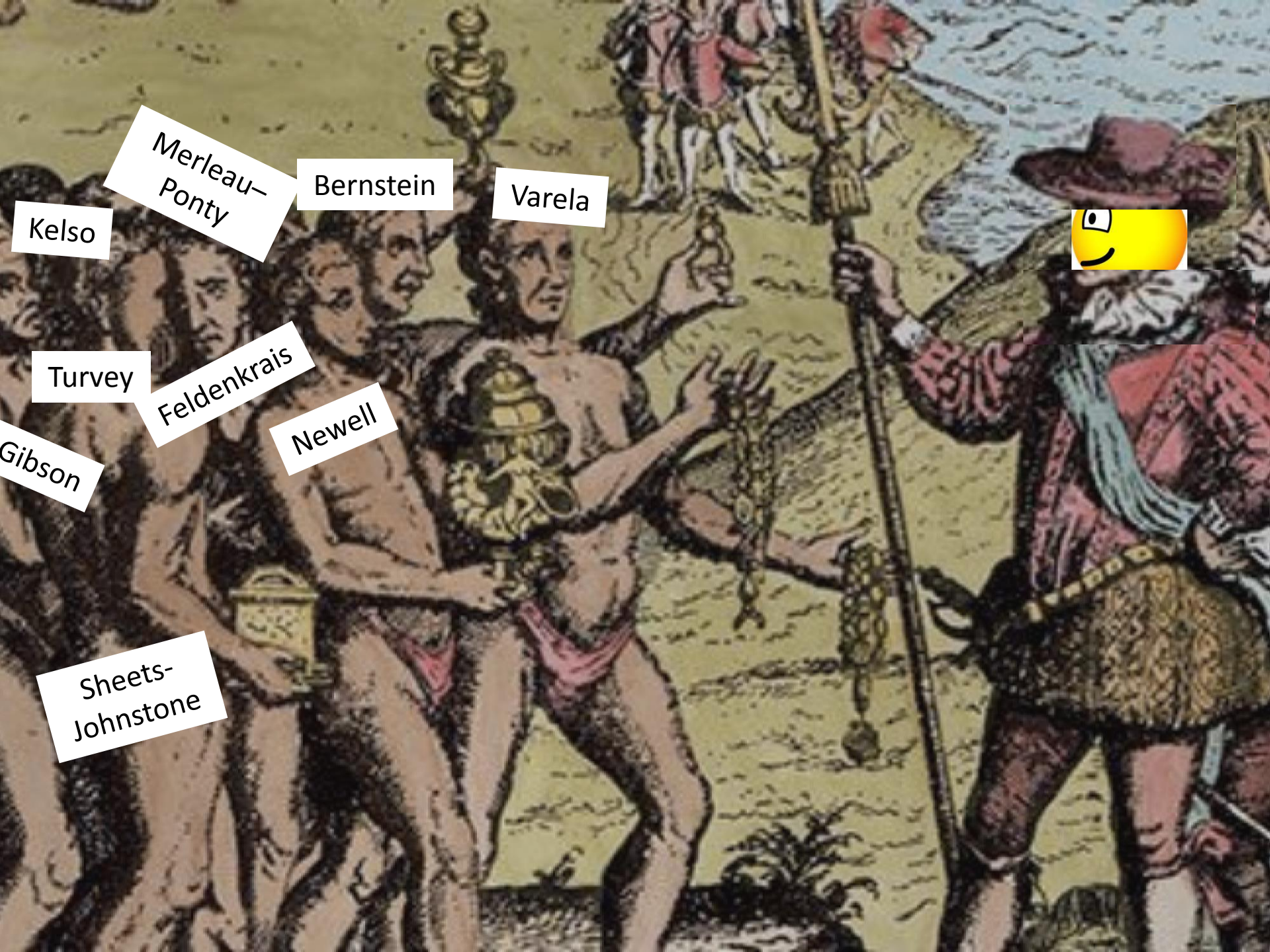
END

OF

THE

WORLD

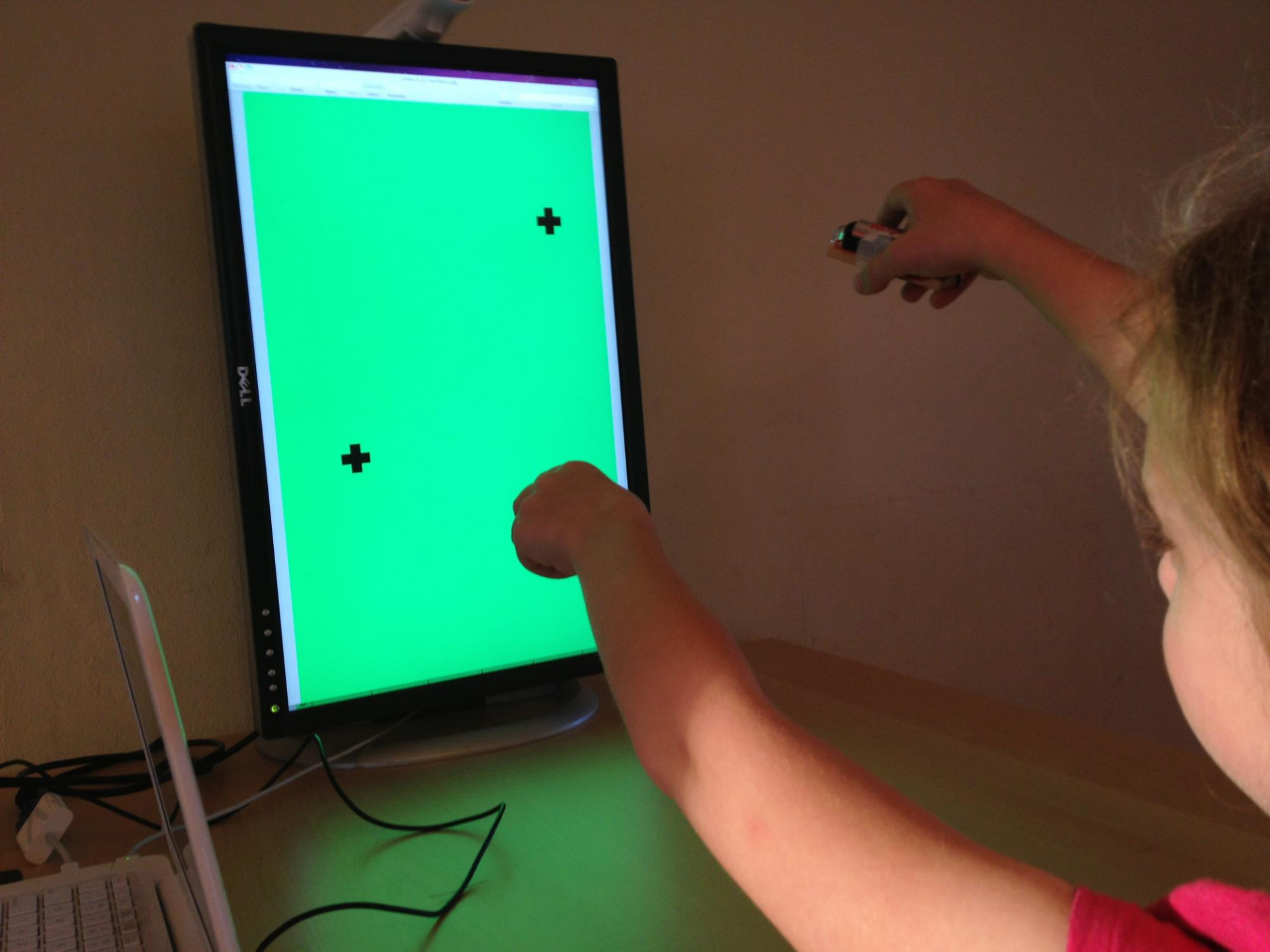



Merleau–
Ponty
Bernstein
Varela
Kelso
Turvey
Feldenkrais
Newell
Gibson
Sheets-
Johnstone

Enactivism

(Varela, Thompson, & Rosch, 1991)

- “In a nutshell, the enactive approach consists of two points:


DELL

Attentional Anchor

A Collaborative Construct

- **Hutto** & **Sánchez-García** (2015)
  Phenomenology and the Cognitive Sciences
- **Hutto**, **Kirchhoff**, & **Abrahamson** (2015)
  Educational Psychology Review
- **Abrahamson**, **Shayan**, **Bakker**, & **van der Schaaf** (2016)
  Human Development
- **Abrahamson**, D., & **Sánchez-García**, R. (2016)
  Journal of the Learning Sciences
- **Abrahamson**, D., & **Bakker**, A. (2016)
  Cognitive Research: Principles & Implications
- **Duijzer**, **Shayan**, **Bakker**, **van der Schaaf**, & **Abrahamson** (2017)
  Frontiers in Psychology
- **Shvarts** & **Abrahamson** (under review)
  American Educational Research Association

Let’s make a circle!

Abrahamson, D., & **Bakker**, A. (2016). Making sense of movement in embodied design for mathematics learning. In N. Newcombe and S. Weisberg (Eds.), Embodied cognition and STEM learning [Special issue]. *Cognitive Research: Principles and Implications, 1*(1), 1-13.

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Movements

- Distal movement
- Proximal movement
- Sensorimotor scheme

Abrahamson, D., & **Bakker**, A. (2016). Making sense of movement in embodied design for mathematics learning. In N. Newcombe and S. Weisberg (Eds.), Embodied cognition and STEM learning [Special issue]. *Cognitive Research: Principles and Implications, 1*(1), 1-13.

Abrahamson, D., & **Shulman**, A. (2017, July).

Constructing movement in mathematics and dance: An interdisciplinary pedagogical dialogue on subjectivity and awareness.

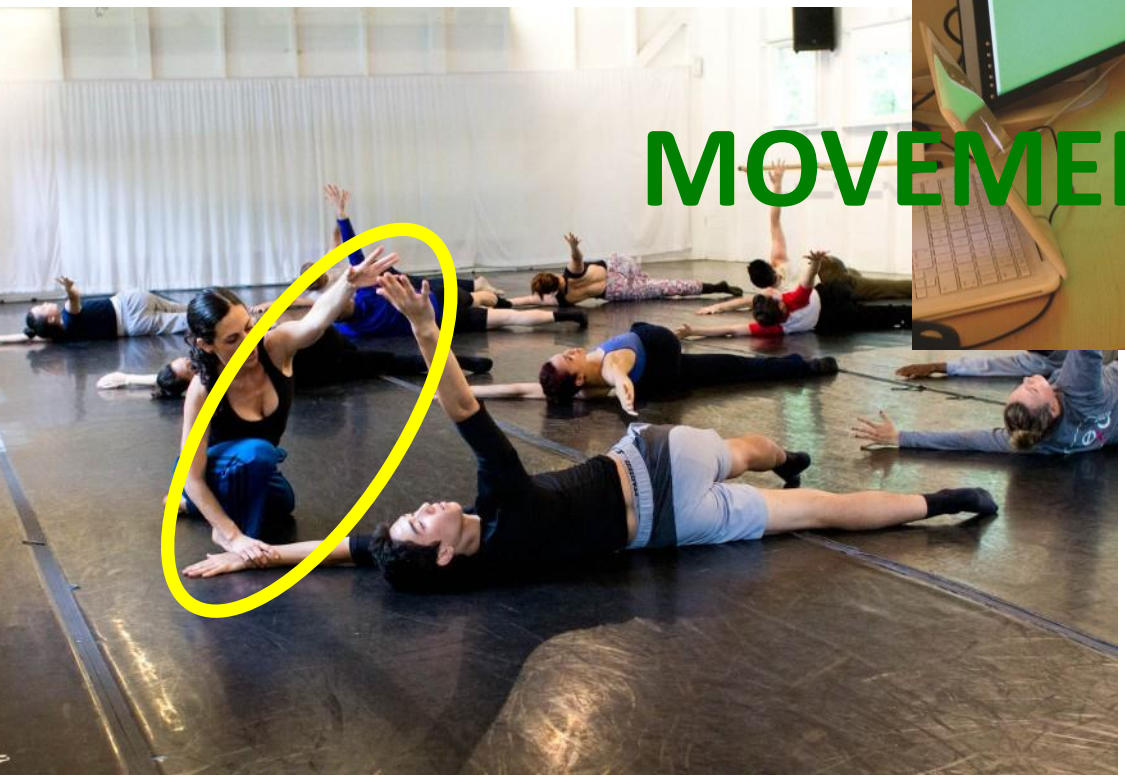
Movement: Brain, Body, Cognition,
Oxford, UK, July 9-11.

www.TinyURL.com/**Dor**Ami2017

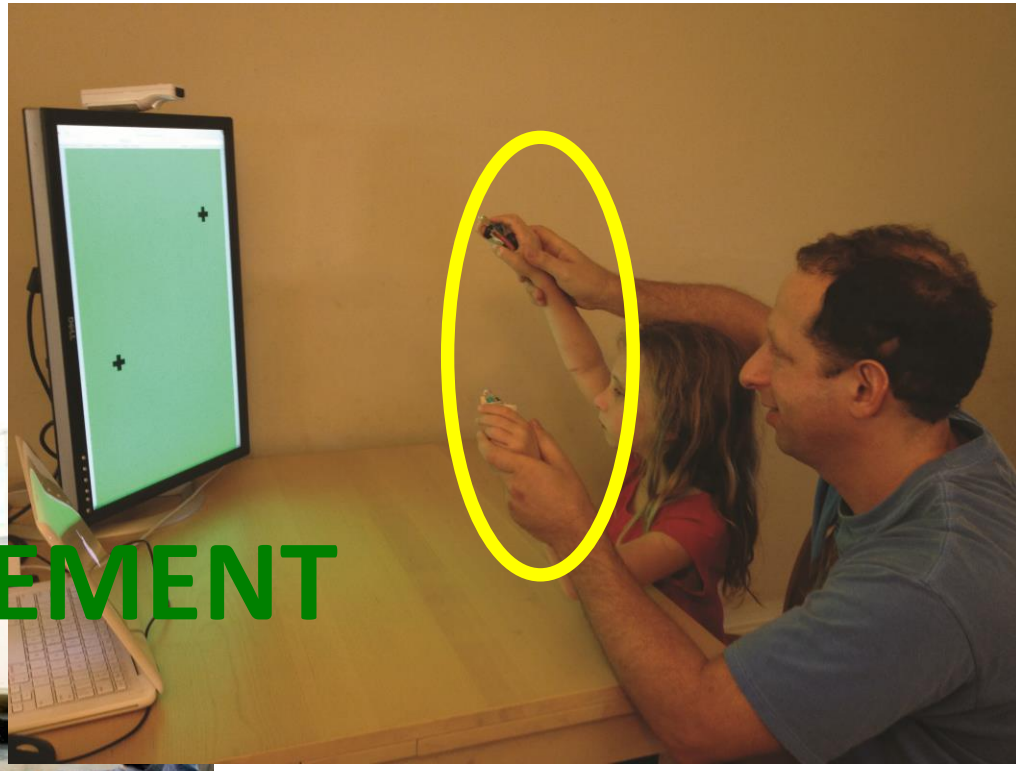
The phenomenon of movement:

- conceptualize phenomenology of movement
- theorize individual awareness of movement
- understand mechanisms of guided movement

DANCE



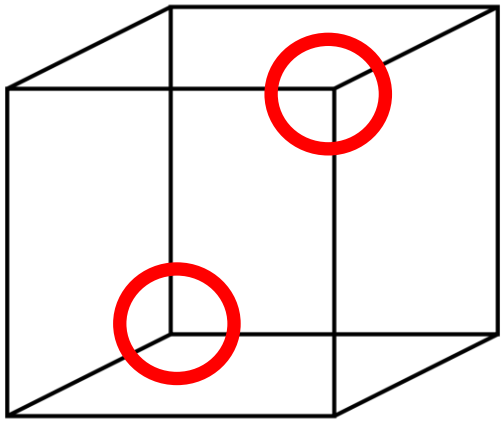
MOVEMENT

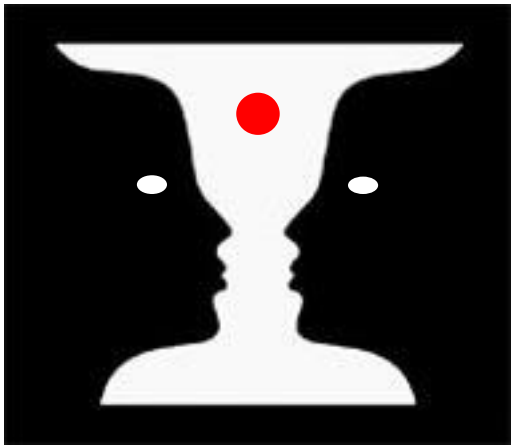


MATH

Intellectual foundations:

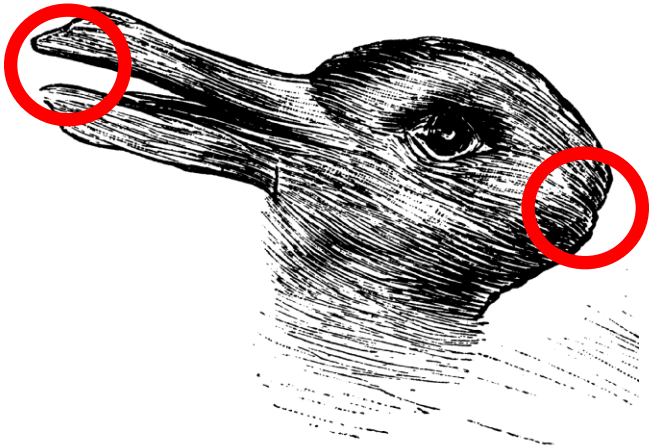
- Feldenkrais Method
- Constructivism
- Enactivism
- Systems Dynamics
- Sociocultural Theory





Perception is…

- *Active (not passive)*
- *Constructed (not inherent)*
- *Relational (not monistic)*
- *Subjective (not objective)*
- *Tacit (not conscious)*



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Thoughts

- A growing conviction that our respective pedagogical worlds have much to share and debate
- Movement as an inherently polysemous ontology
- When mathematics learning is conceptualized as founded on sensorimotor entrainment, we share in pedagogical practices
  - center on subjective idiosyncratic phenomenology of movement
  - attempt to stimulate awareness of orientation so that students modify the action to accord with a normative or desired cultural practice

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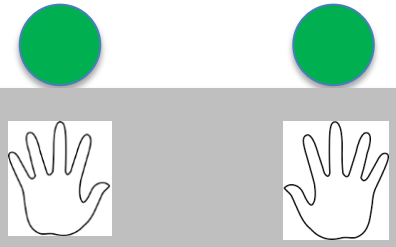
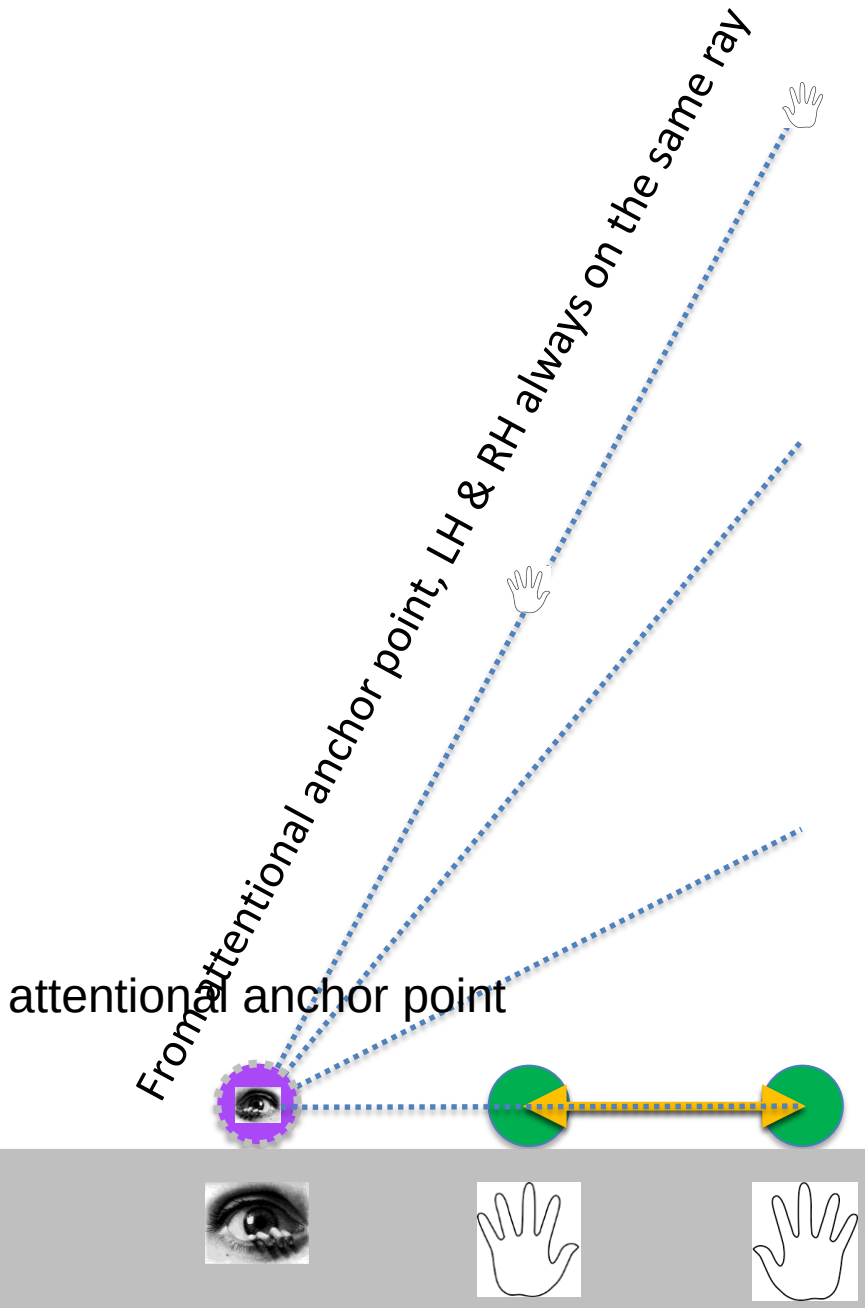
13 Ways of Looking at Two Hands Moving

A Pairs Game

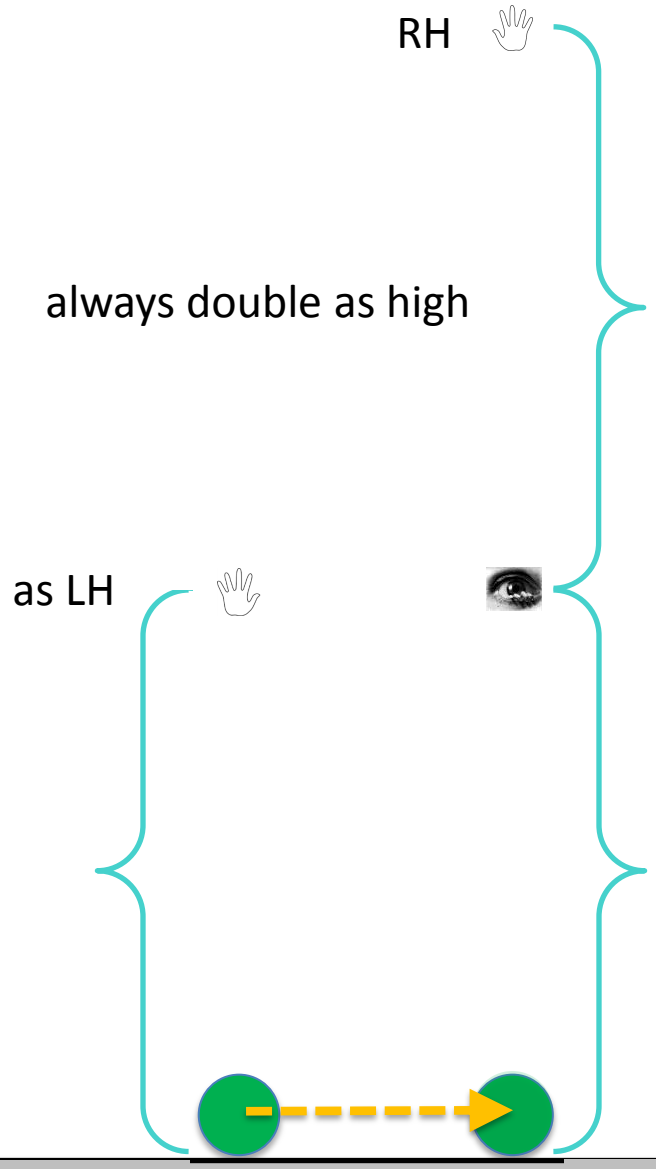
On the Polysemous Meanings of Movement



A


From attentional anchor point, LH & RH always on the same ray
attentional anchor point

B



Your Mission, If You Choose to Accept It

- You have received your secret strategy
- Sit opposite your buddy, within reach
- Mirroring each other, enact the movement, each according to *your own* strategy
- Try to guess the other person's strategy
  - You can hint, but no talking
- Perform A's instructions *together* (4 hands), adjusting RH/LH so you are **mirroring** each other
- Now play it 2 hands (one hand each), only RH

Debrief

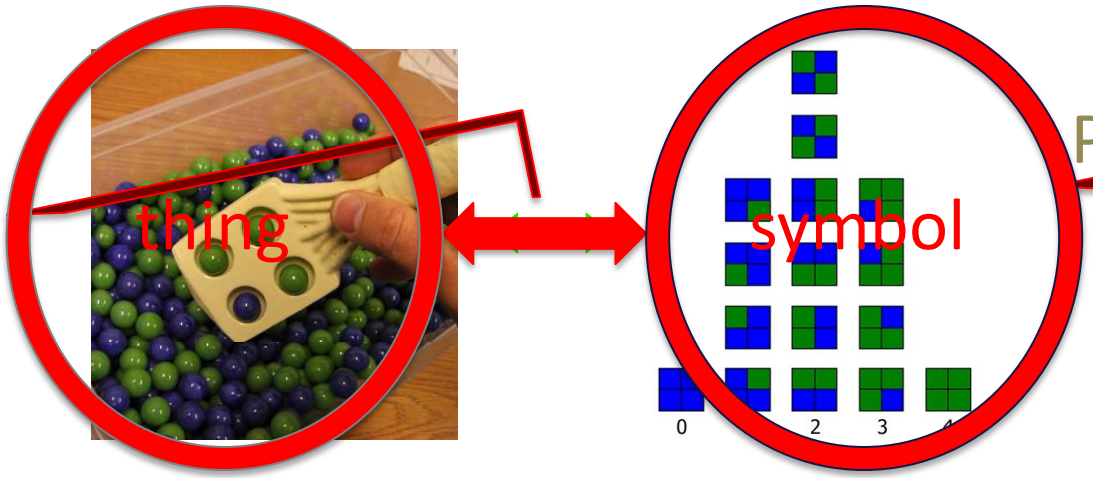
- How successful were you in guessing the other's strategy?
- In particular, what were you doing to figure this out?
- What did you do in order to help the other person figure out how you were moving?
- How did you coordinate the mirroring?
- How did you coordinate the distributed co-enacting (one hand each)?
- What might be the relevance of this exercise for a research program to understand how people reason mathematically? How about design for learning?
- Can you think of a different concepts / movements that could be approached this way?
- What else, if anything, was interesting for you in this exercise?

Embodied Design

(Abrahamson, 2009, 2015)

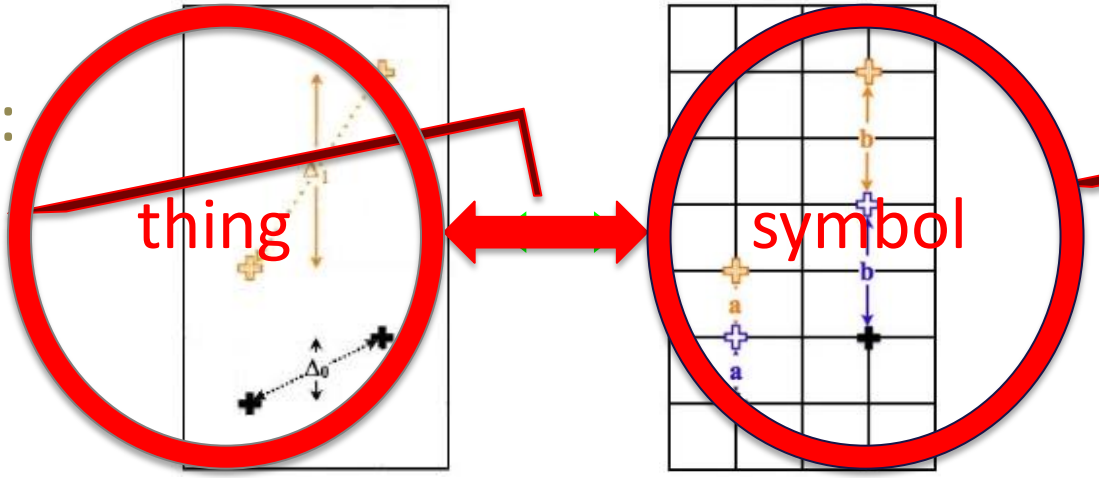
…is a pedagogical framework that seeks to promote grounded conceptual learning by creating situations in which students can be guided to negotiate tacit and cultural ways of perceiving and acting.

Appropriating Mathematical Forms



Perception-based design:

Action-based design:






Anne-Ciska Cuiper Marijke Veugen Loes Boven Carolien Duijzer

COLLABORATING RESEARCHERS AT UTRECHT UNIVERSITY

Rosa Alberto Shakila Shayan Arthur Bakker Marieke van der Schaaf




Action-Based Embodied Design: The Mathematics Imagery Trainer

- The design simulates naturalistic conditions for conceptual learning. The Mathematics Imagery Trainer creates a field of promoted action fostering the practice of culturally valued ways of moving.
- The designer instantiates a mathematical concept in the form of an information structure (system of functions).
- This information structure is programmed into a technological device as procedural rules.
- The information structure is latent to this device and becomes apparent and known to a person only through interacting with the device and reflecting on the interaction.
- The interaction is at an interface: a natural-user interface for human-computer interaction.
- The person interacts with the device by enacting physical movements in an attempt to satisfy prescribed task demands. The information structure privileges certain actions — physical postures and therefore movement regimes — over others as satisfying task demands; this bias is surfaced at the interface via rule-based real-time feedback on performance.
- The person discovers, constructs, regulates, and refines their own subjective means of enacting these movements, theorized as the emergence of a new sensorimotor scheme.
- The scheme is evident for observation as fluent information-invariant transformations of the environment.
- This scheme incorporates a new sensory structure orienting and coordinating the motor activity. We call this new sensory structure an attentional anchor (Hutto/Sánchez-García).
- The attentional anchor is the sensory component of the new affordance that the person comes to apprehend in the field of promoted action, that is, when engaging the technological device in the context of this task-based activity, and perhaps beyond.
- AA as self-imposed environmental constraint (“given that”) ; solution assimilated as task constraints (“even as I”).
- One might say that the new sensorimotor scheme, including the new attentional anchor, are the person's enactive incorporation of the design’s latent information structure.

Dor Abrahamson

Design-Based Research

- An approach to educational research, wherein practice of educational design serves as context for pursuing problems of importance to the science of learning.
- Research problems emerge where our intuition fails, the literature does not provide answers, and our colleagues are stumped. This happens during:
  - Ideation
  - Engineering
  - Data gathering
  - Analysis

Embodied Design

(Abrahamson, 2009, 2015)

- An approach to educational design-based research that is both informed by embodiment theory and informing the evaluation and development of this theory
- Two design genres
  - perception-based design
  - action-based design

Embodied Design:

Constructing Means for Constructing Meaning

- General Design Objective
  - Learners will accept mathematical forms as meaningful ways of thinking and acting
- Why do they accept the forms?
  - Perception-based design: *Inferential Parity*
  - Action-based design: *Functional Parity*
- What does parity do?
  - Parity is epistemic grounds for appropriating mathematical signification of embodied skill

Embodied Design as a Two-Step: Elicit Tacit Schema, Signify It Culturally

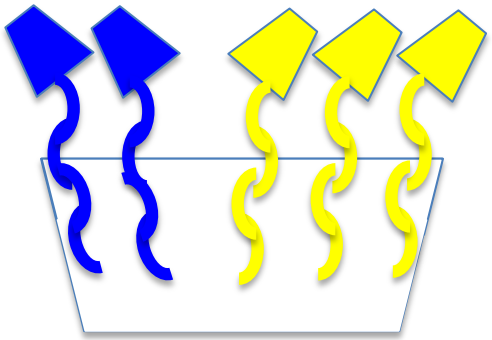
- Elicit Subjective Meanings
  - Pose perceptuo/motor problem
    - perceptual judgment
    - motor action
  - Encourage articulation
- Signify Subjective Meanings
  - Introduce new affordances:
    media, frames of reference, symbolic artifacts for enacting, explaining, evaluating
  - Encourage coordination with grounding meanings

“The great advances in science usually result from new tools rather than from new doctrines.”

Freeman Dyson, 1996

Mathematics Imagery Trainer #1

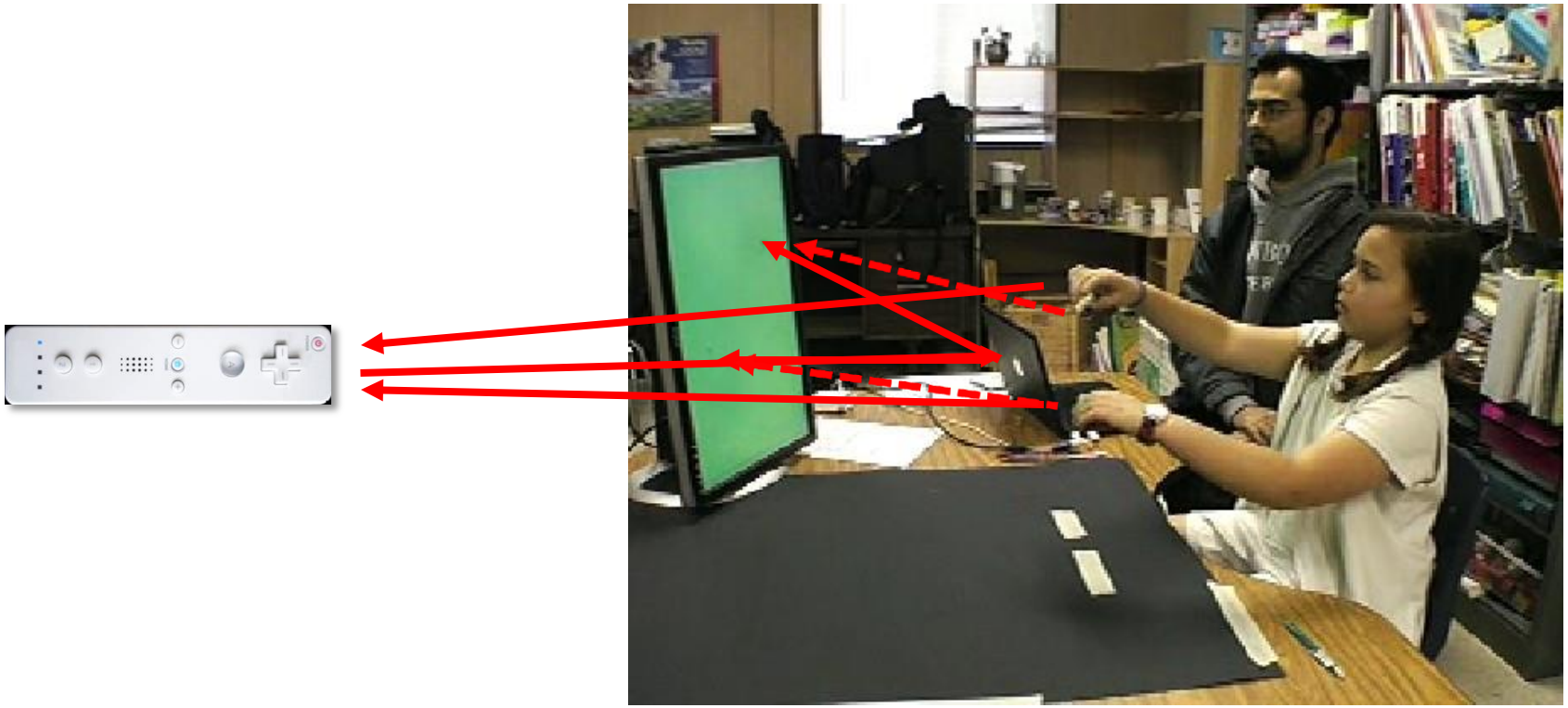
2:3 = 4:6



In search of an embodied design for proportional equivalence

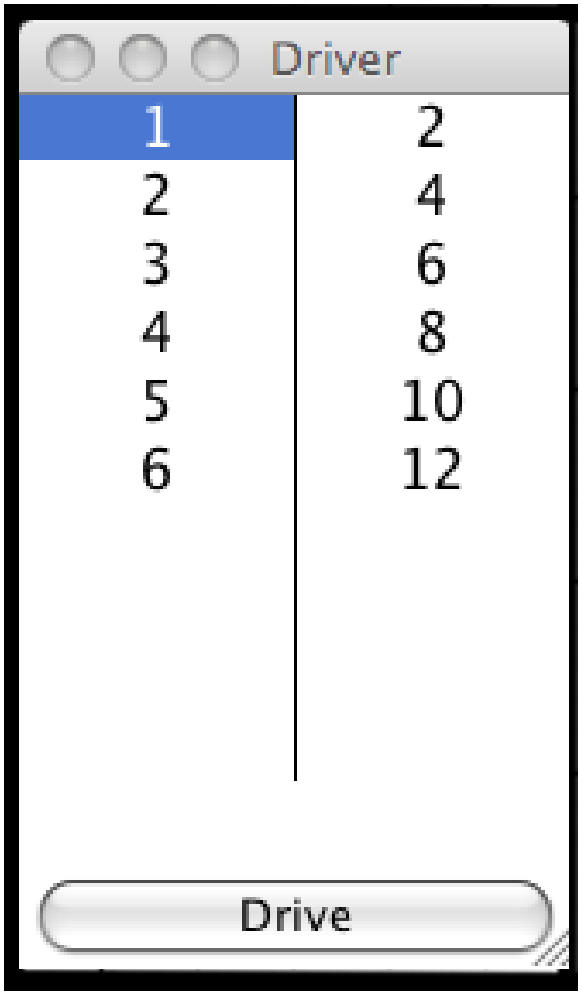
Johnny Chung Lee

Mathematics Imagery Trainer #2

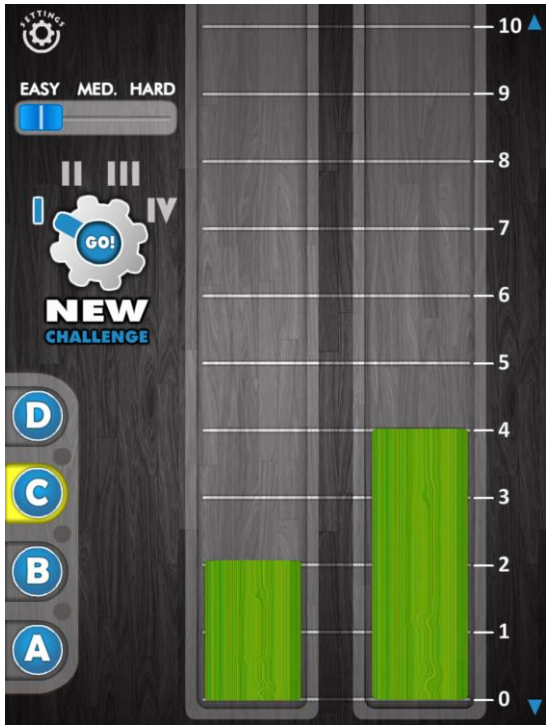


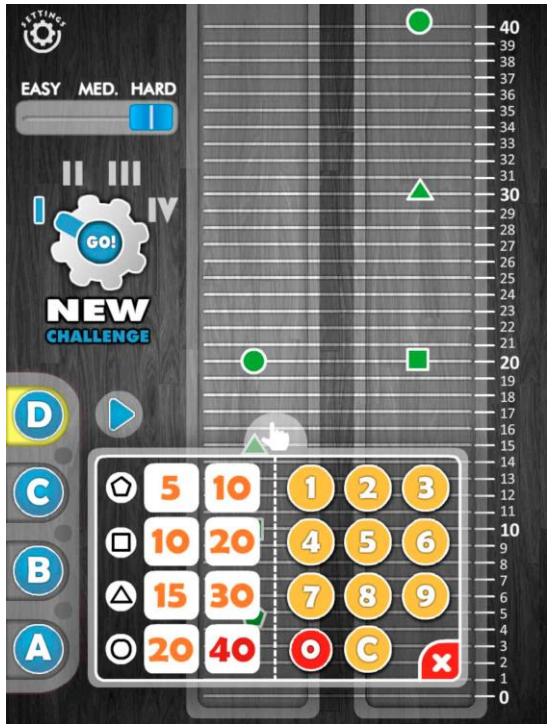
Reinholz, D., **Trninic**, D., **Howison**, M., & **Abrahamson**, D. (2010). It's not easy being green: embodied artifacts and the guided emergence of mathematical meaning. In *Proc. of PME-NA 32* (Vol. VI, Ch. 18: Technology, pp. 1488 – 1496). Columbus, OH: PME-NA.

Enter Symbolic Artifacts



MIT-P for iPad





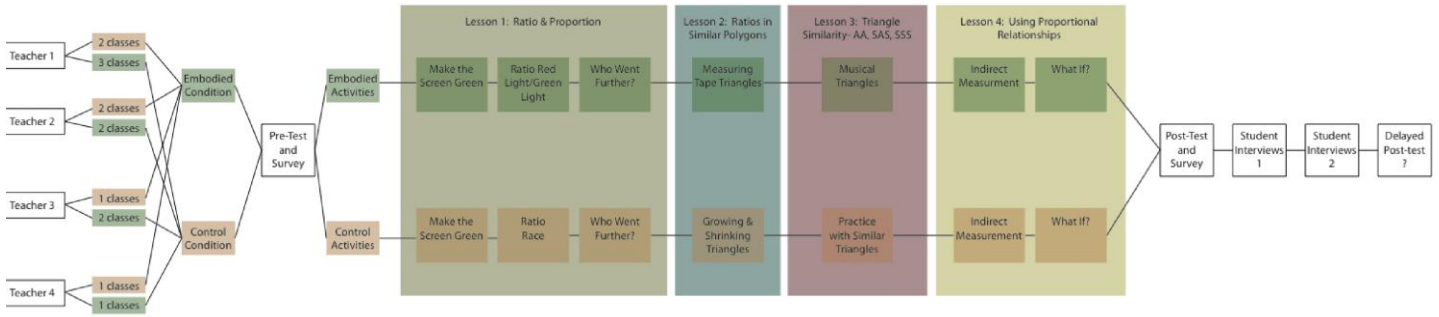
tinyurl.com/FreeMITP

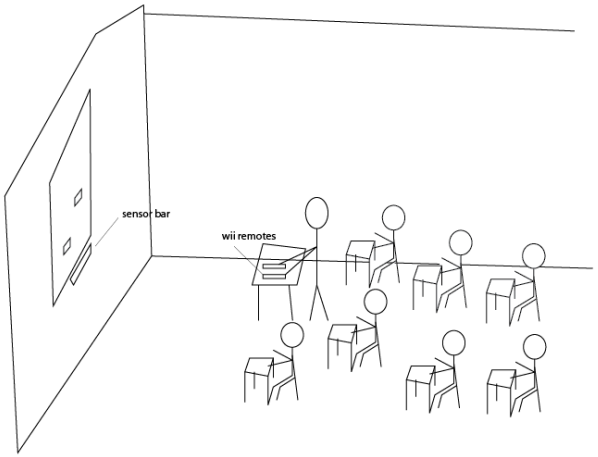
MIT-P for iPad

Negrete, A. G., **Lee**, R. G., & **Abrahamson**, D. (2013). Facilitating discovery learning in the tablet era: rethinking activity sequences vis-à-vis digital practices. In *Proc. of PME-NA 35)* (Vol. 10: "Technology," p. 1205). Chicago, IL: University of Illinois at Chicago.

tinyurl.com/FreeMITP

Carmen Petrick dissertation





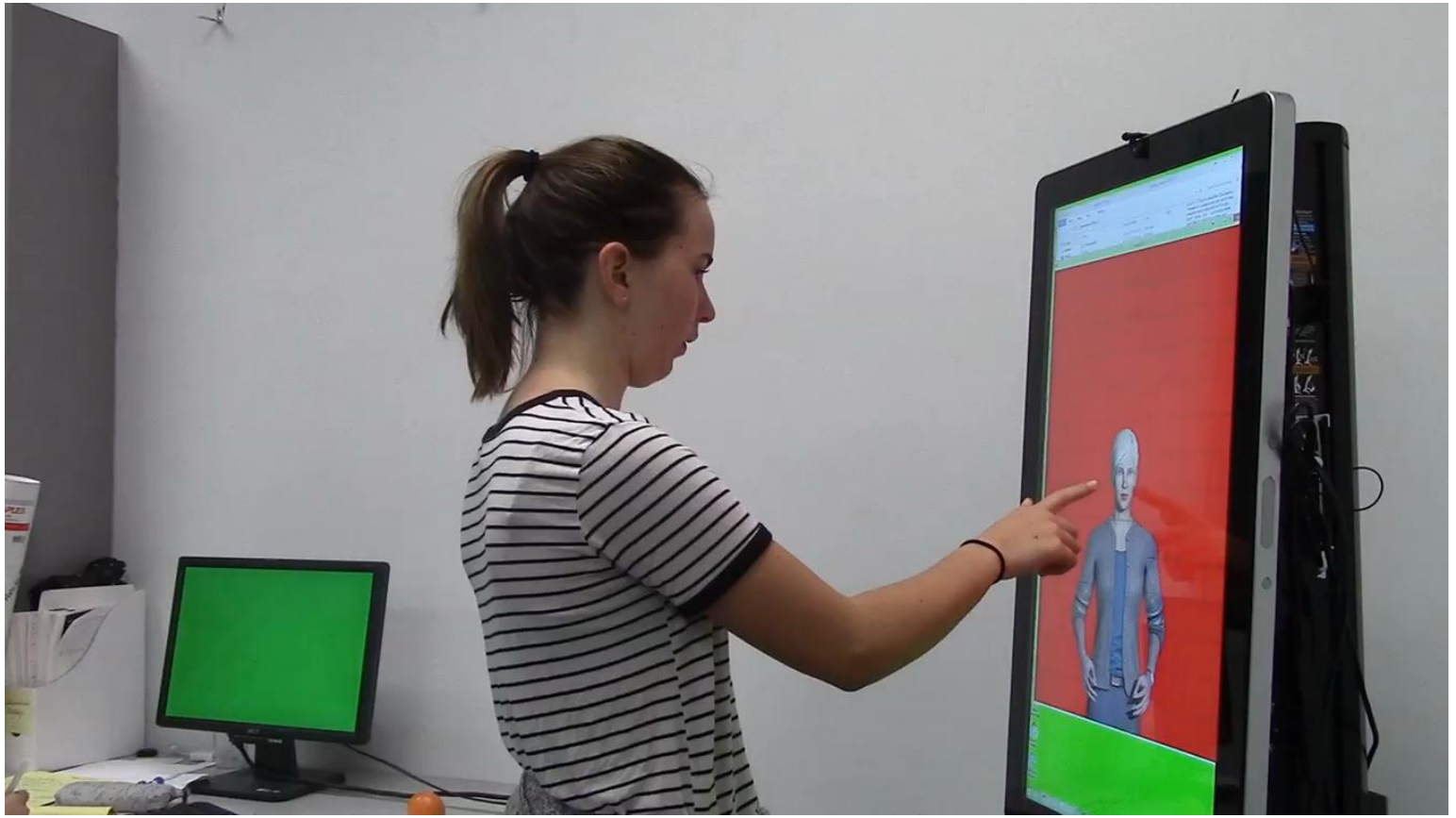
Petrick, C. J. (2012). *Every body move: Learning mathematics through embodied actions*. Austin, TX: University of Texas at Austin. (Unpublished doctoral dissertation).


USE

Gesture Enhancement of Virtual-Agents Mathematics Tutors

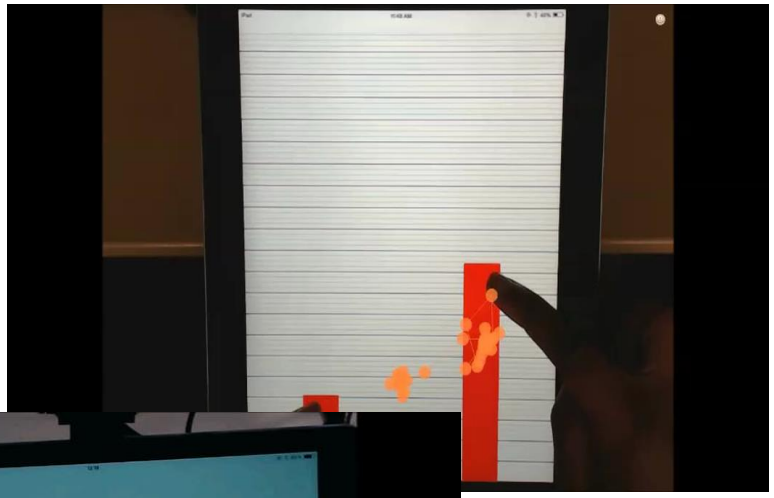
Flood, V. J., **Schneider**, A., & **Abrahamson**, D. (2014). Gesture enhancement of a virtual tutor via investigating human tutor discursive strategies: forms and functions for proportions. In *Proc. of ICLS* (Vol. 3, pp. 1593-1594). Boulder, CO: International Society of the Learning Sciences.

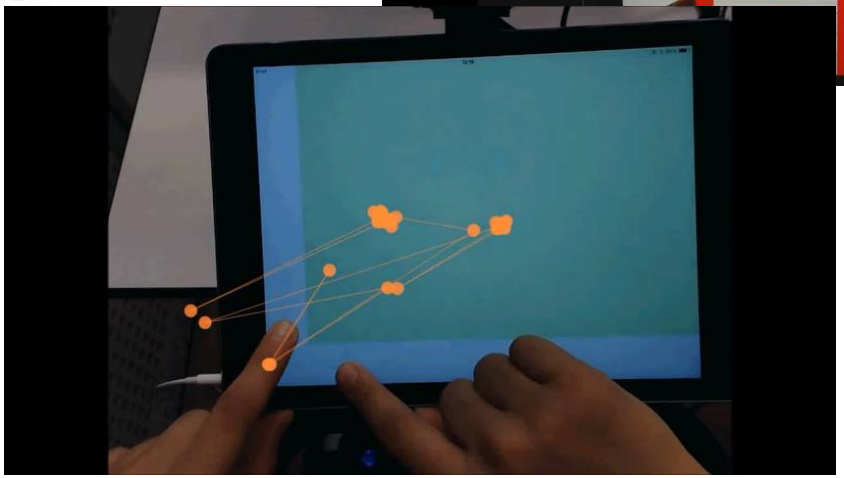
Gesture Enhancement of Virtual-Agents Mathematics Tutors



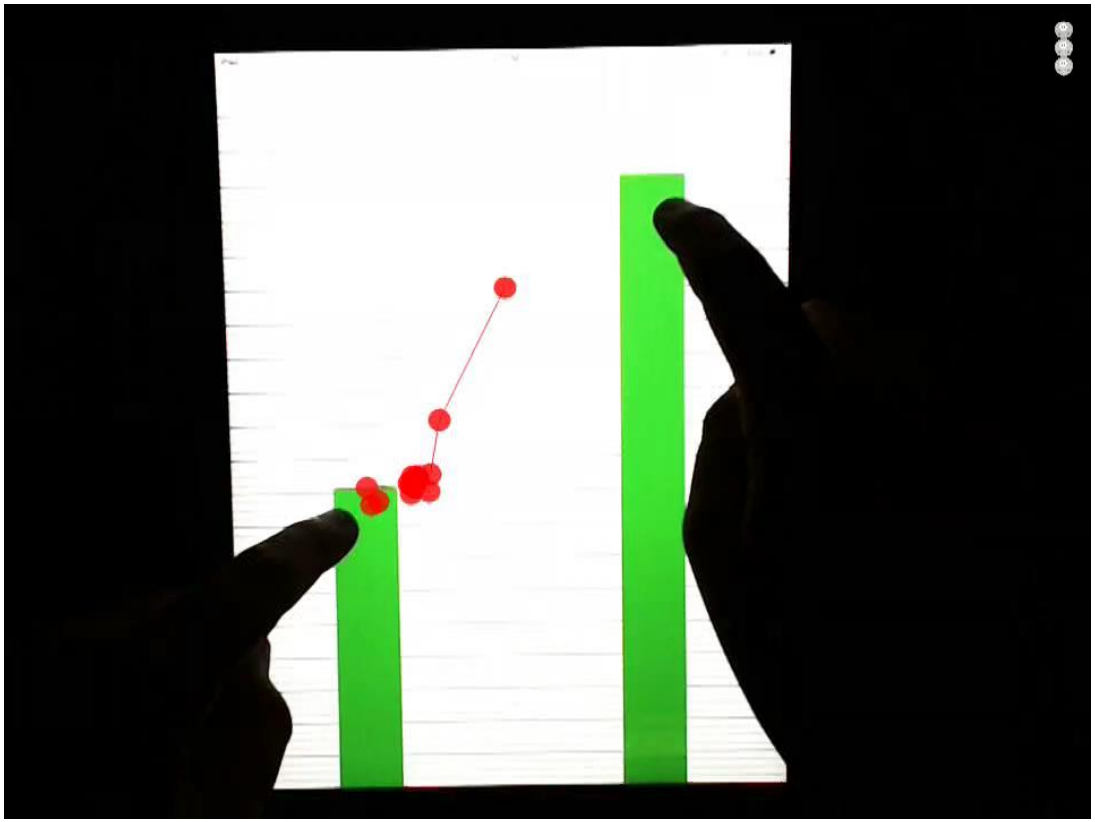
Abdullah, A., **Adil**, M., **Rosenbaum**, L., **Clemmons**, M., **Shah**, M., **Abrahamson**, D., & **Neff**, M. (2017). Pedagogical agents to support embodied, discovery-based learning. In the *Proceedings of the 17th Int. Conference on Intelligent Virtual Agents (IVA 2017)* (pp. 1-14). Cham: Springer.

Meanwhile, in Utrecht…



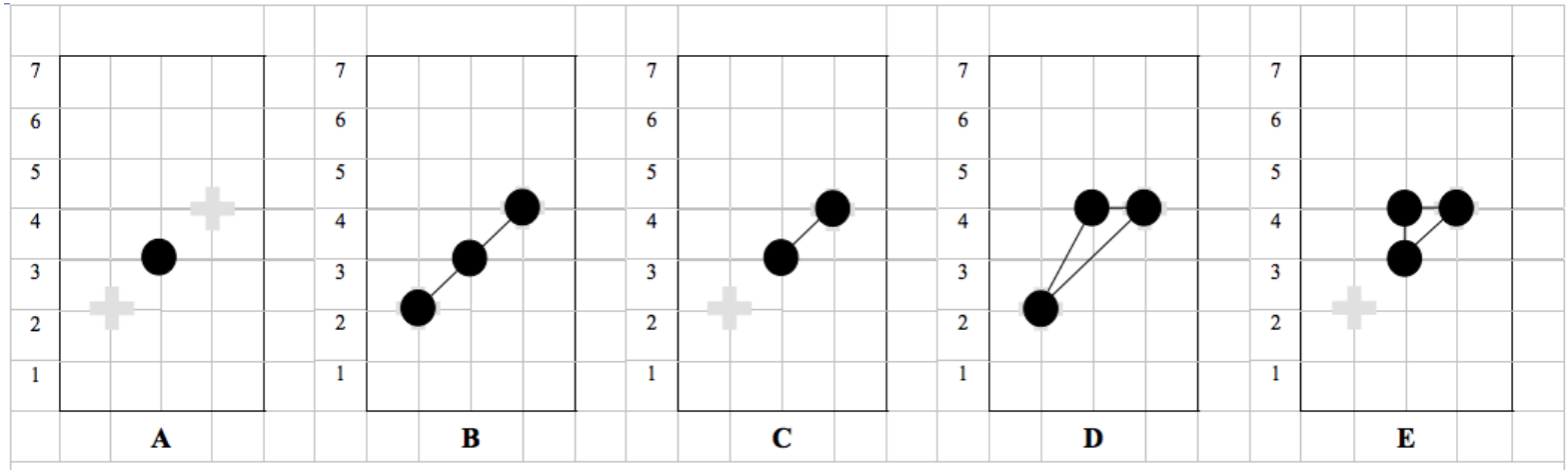


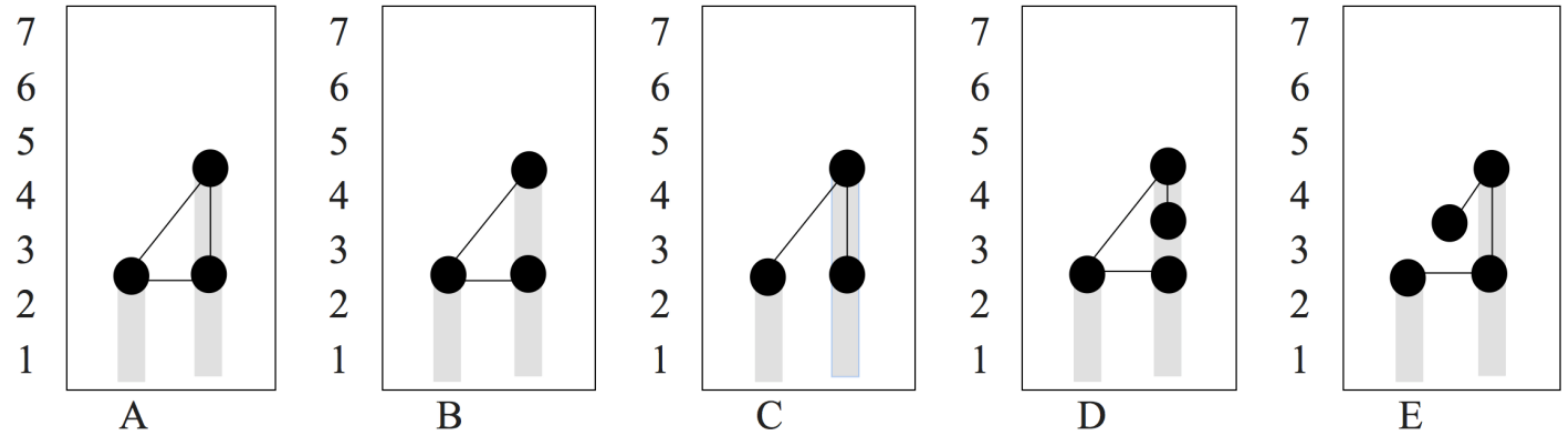
Eye-Tracking Study: Parallel



Shayan, S., **Bakker**, A., **Abrahamson**, D., **Duijzer**, C., & **van der Schaaf**, M. (2015). The emergence of proportional reasoning from embodied interaction with a tablet application: an eye-tracking study. In L. Gómez Chova, A. López Martínez, & I. Candel Torres (Eds.), *Proceedings of the 9th International Technology, Education, and Development Conference (INTED 2015)* (pp. 5732-5741). Madrid: IATED.

Typical Eye-Tracking Patterns





Original Paper

Human Development 2015;58:218–244
DOI: 10.1159/000443153

Human Development

Eye-Tracking Piaget: Capturing the Emergence of Attentional Anchors in the Coordination of Proportional Motor Action

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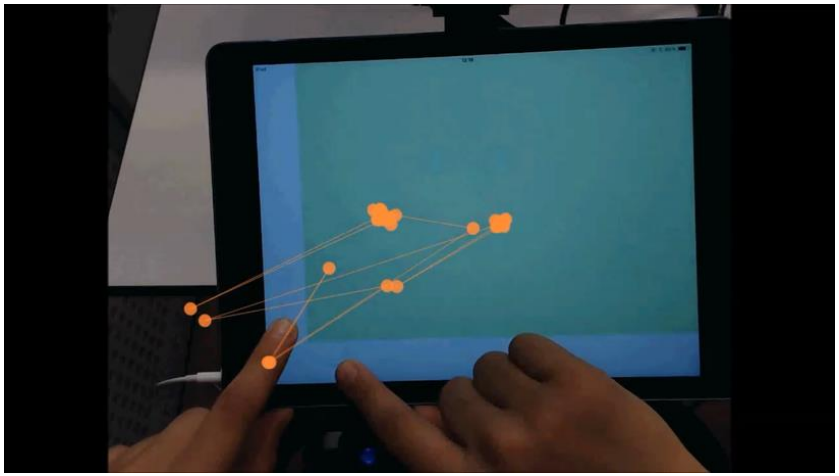
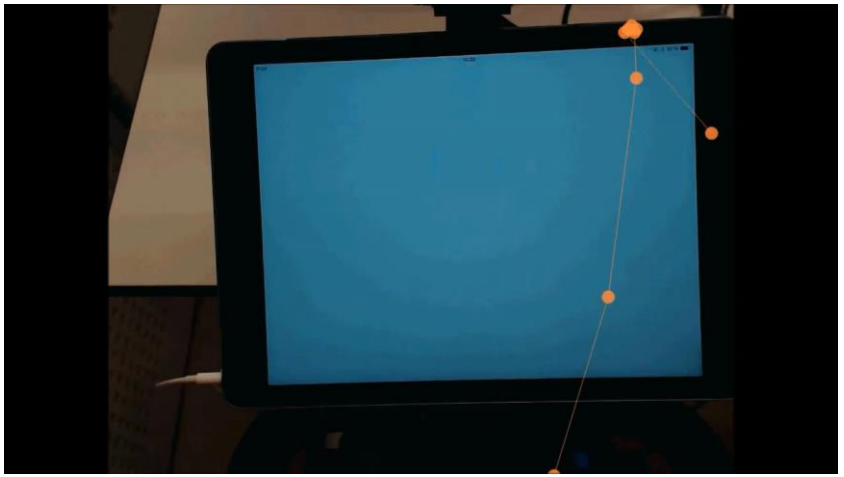
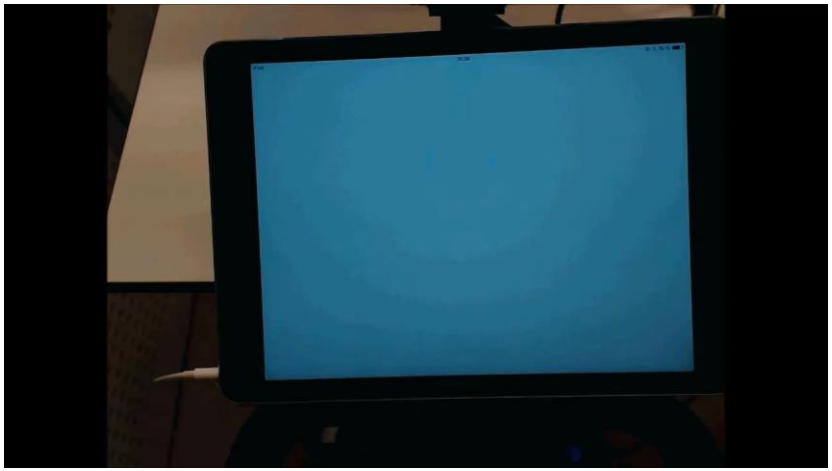
Marieke van der Schaaf[b]

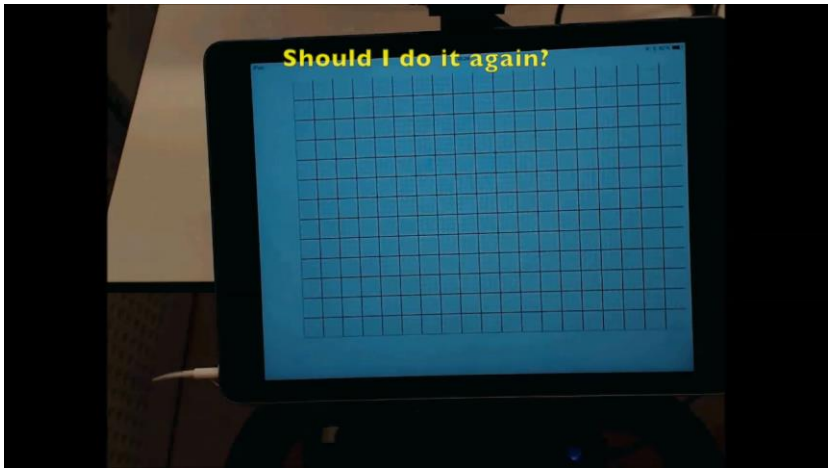
[a]University of California, Berkeley, Calif., USA; [b]Utrecht University, Utrecht, The Netherlands

Eye-Tracking Study: Orthogonals

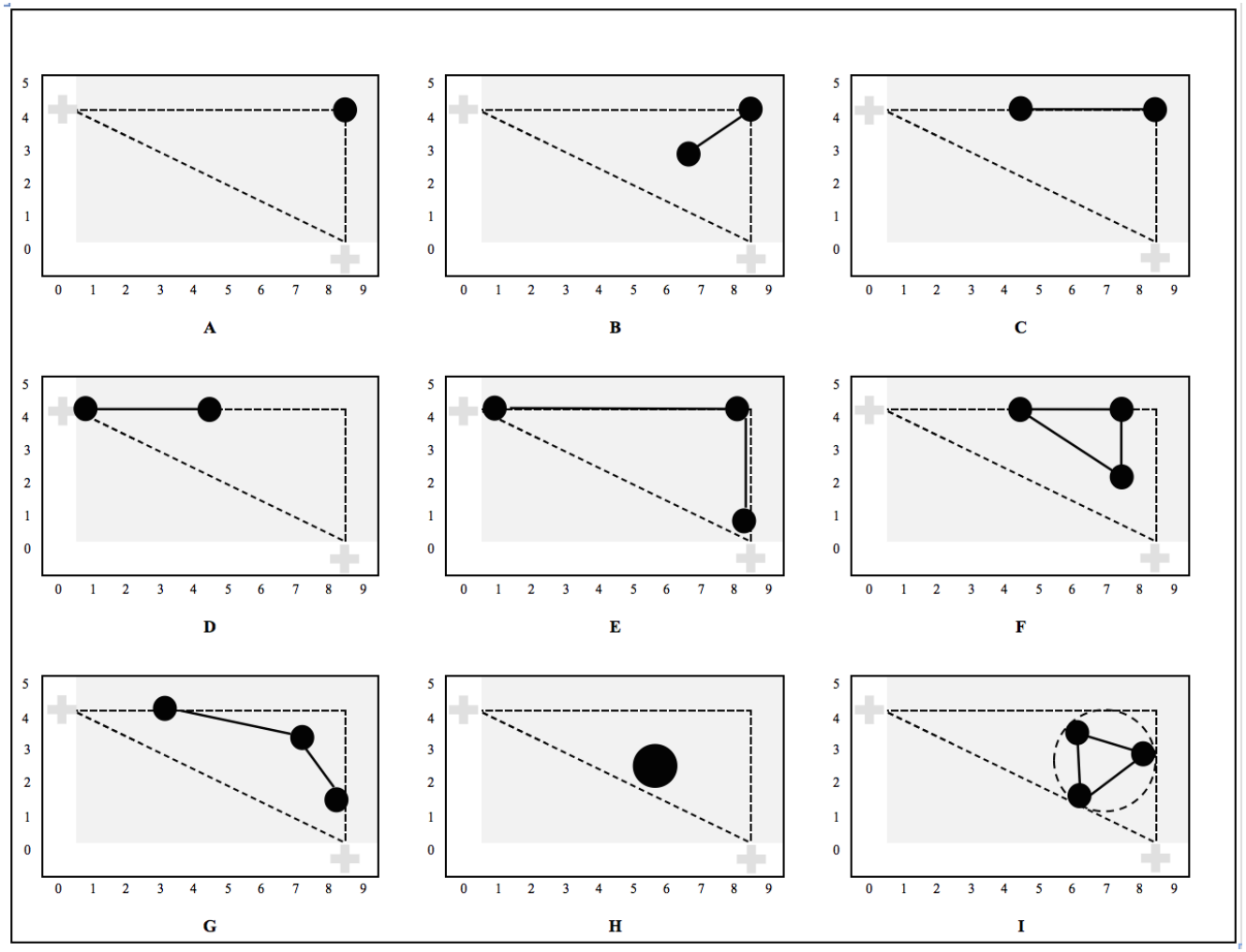




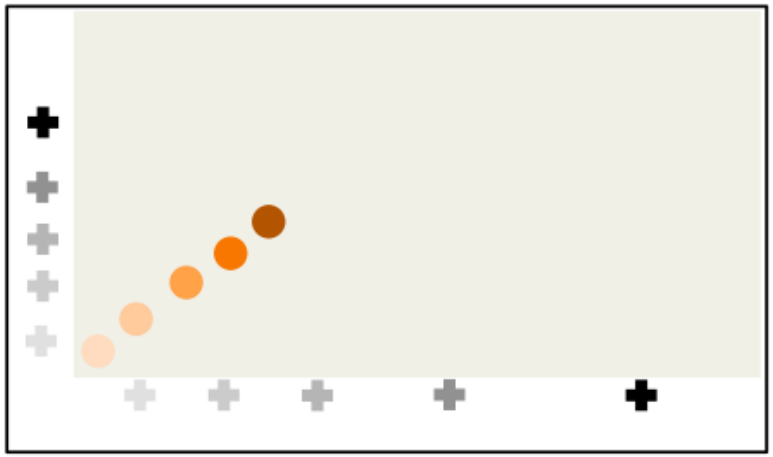


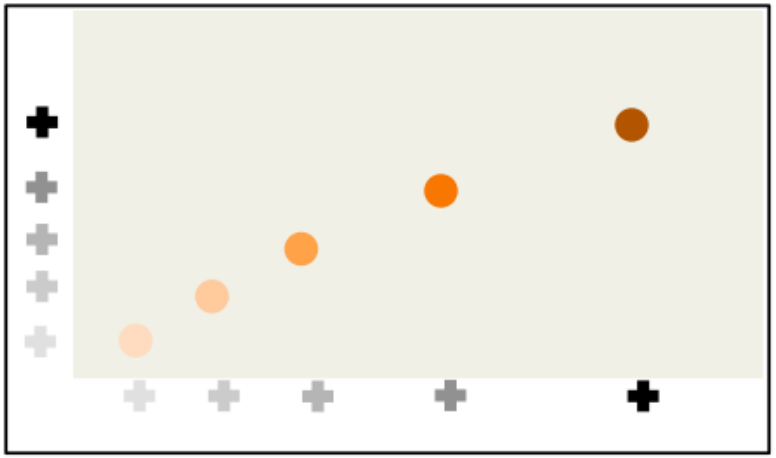


Typical Eye-Tracking Patterns



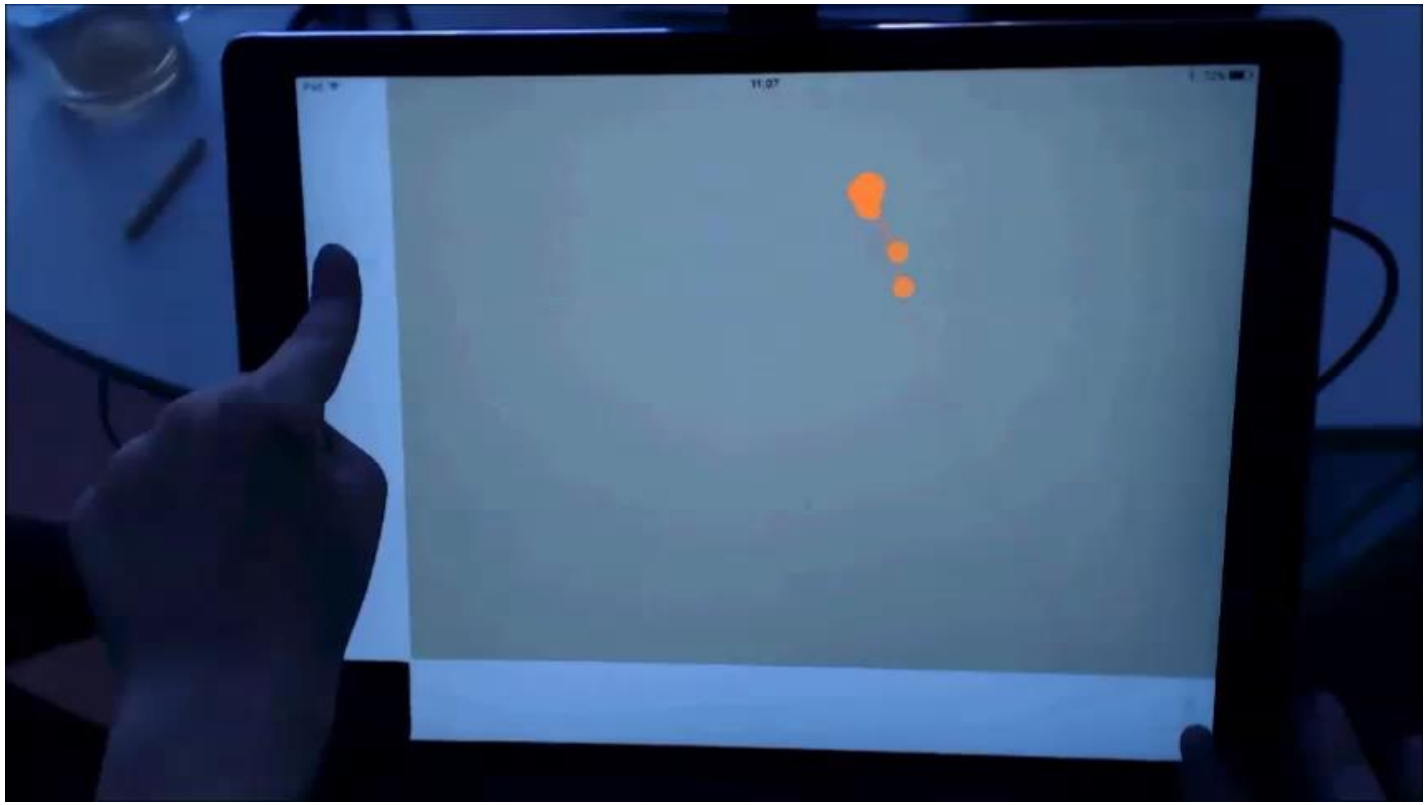
Dynamic Visualization Overlay From ET Teaching to ET *in* Teaching?



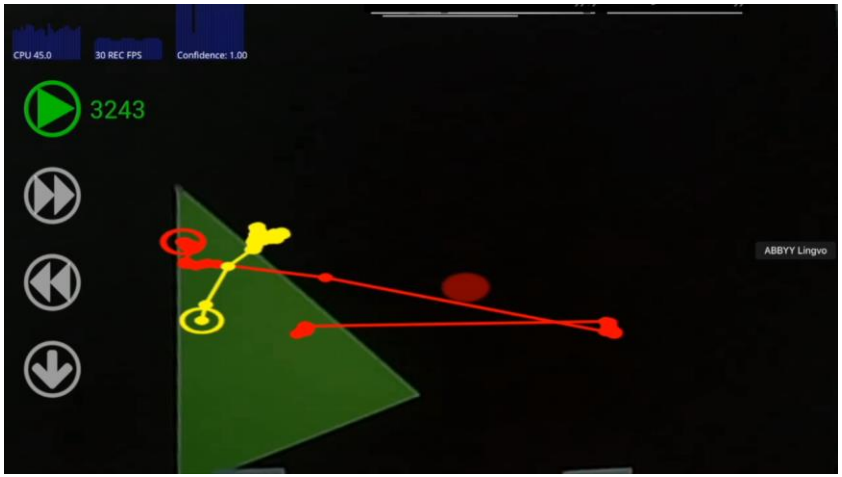


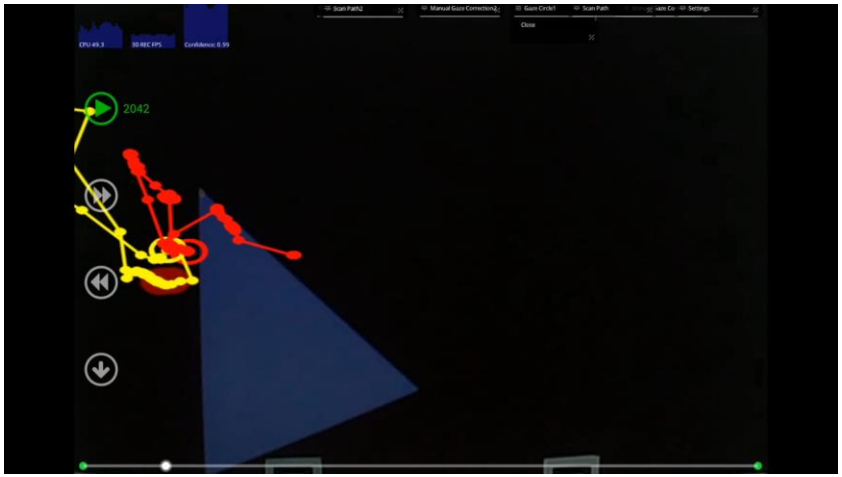
Abrahamson, D., **Shayan**, S., **Bakker**, A., & **van der Schaaf**, M. F. (2016). Eye-tracking Piaget: Capturing the emergence of attentional anchors in the coordination of proportional motor action. *Human Development*, *58*(4-5), 218-244.

When Teachers Know a Task's Attentional Anchors



DUET: Dual Eye Tracking





Shvarts, A. (in press). Joint attention in resolving the ambiguity of different presentations: A dual eye-tracking study of the teaching-learning process. In N. Presmeg, L. Radford, W.-M. Roth, & G. Kadunz (Eds.), *Signs of signification: Semiotics in mathematics education research*. Dordrecht: Springer.

Shvarts, A., & **Abrahamson**, D. (under review). *Towards a complex systems model of enculturation: A dual eye-tracking study*. American Educational Research Association.

Attentional Anchor

A Collaborative Construct

- **Hutto** & **Sánchez-García** (2015)
  Phenomenology and the Cognitive Sciences
- **Hutto**, **Kirchhoff**, & **Abrahamson** (2015)
  Educational Psychology Review
- **Abrahamson**, **Shayan**, **Bakker**, & **van der Schaaf** (2016)
  Human Development
- **Abrahamson**, D., & **Sánchez-García**, R. (2016)
  Journal of the Learning Sciences
- **Abrahamson**, D., & **Bakker**, A. (2016)
  Cognitive Research: Principles & Implications
- **Duijzer**, **Shayan**, **Bakker**, **van der Schaaf**, & **Abrahamson** (2017)
  Frontiers in Psychology

Thank You

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http://edrl.berkeley.edu

Dynamic-Systems Approach to the Development of Cognition and Action

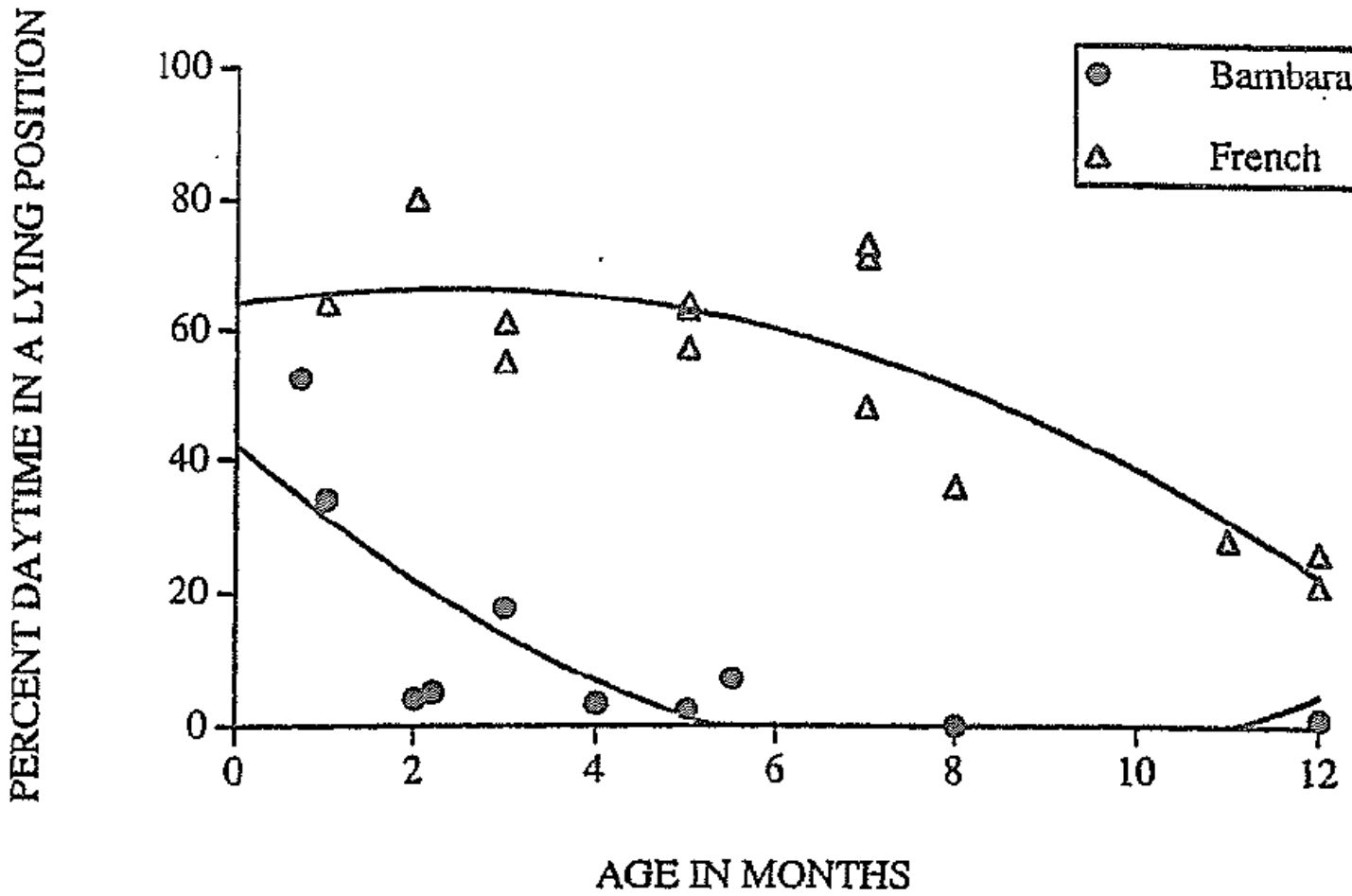
Esther Thelen
(1941-2004)

A-not-B task

Thelen, E., & **Smith**, L. B. (1994). *A dynamic systems approach to the development of cognition and action*. Cambridge, MA: MIT Press.

Fields of Promoted Action

STRETCHING



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Reed, E. S., & **Bril**, B. (1996). The primacy of action in development. In M. L. Latash & M. T. Turvey (Eds.), *Dexterity and its development* (pp. 431-451). Mahwah, NJ: LEA.

Try This at Home!

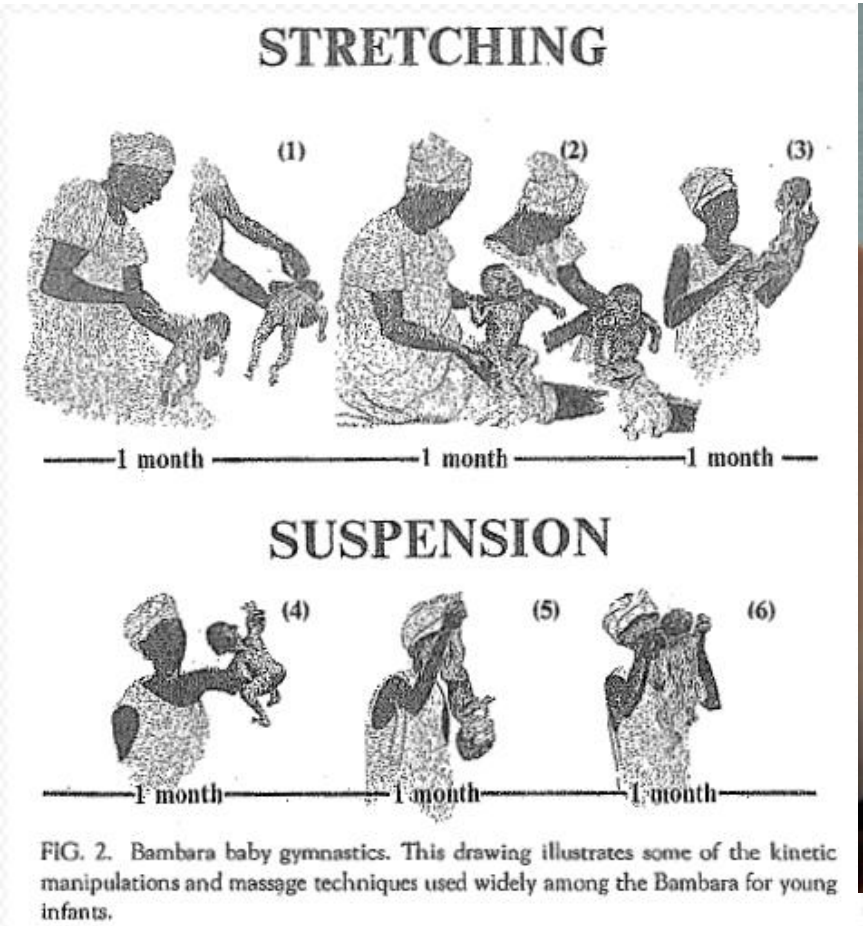


FIG. 2. Bambara baby gymnastics. This drawing illustrates some of the kinetic manipulations and massage techniques used widely among the Bambara for young infants.

Chow, J. Y., **Davids**, K., **Button**, C., **Shuttleworth**, R., **Renshaw**, I. , & **Araújo**, D. (2007). The role of nonlinear pedagogy in physical education. *Review of Educational Research, 77*(3), 251-278.

Interim Summary

1. The body is not a mere input/output device – the body is not just an executioner of disembodied cognitive inferences and directives: the body plays an intrinsic role in cognition – cognition is embodied.

2. The body is in an environment, an ecology with built-in constraints, e.g., its own biomechanics, or gravity, and these constraints create for the agent emergent problems to solve; the solution of these problems, if practiced repeatedly and under varying conditions, can become part of the agent's operational repertory, e.g., walking.

3. Humans can create specific conditions for other humans to experience and solve problems, so that those younger humans are acculturated to participate in vital practices.

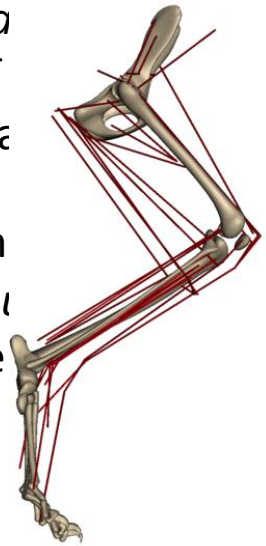
Repetition Without Repetition

Nikolai Bernstein
(1896-1966)

"Repetitions of a movement or action are necessary in order to *solve a motor problem* many times (better and better) and *to find the best ways* of solving it."

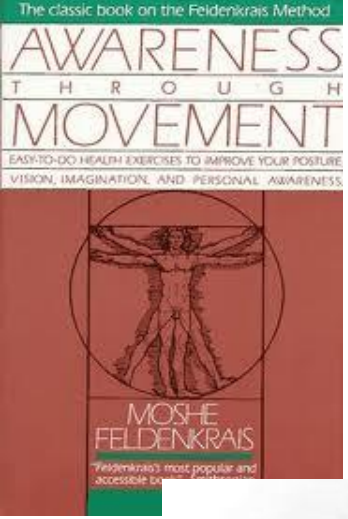
"…[D]uring a correctly organized exercise a student is repeating many times, *not the mea* a given motor problem, but *the process of* ne changing and improving of the mea

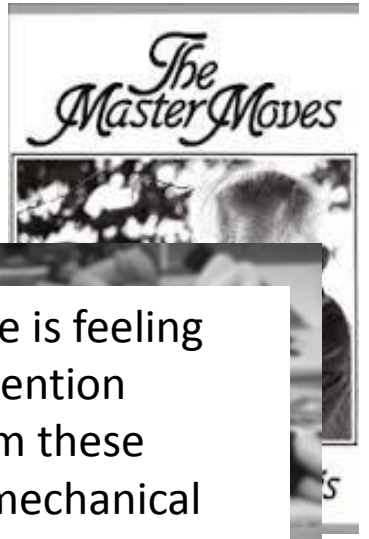
"The fact that the 'secrets' of swim g *are not in some special body movements bu ensations and corrections* explains why these npossible to teach by demonstration "



Bernstein, N. A. (1996). *Dexterity and its development* (M. L. Latash & M. T. Turvey, Eds.). Mahwah, NJ: Lawrence Erlbaum Associates.

Functional Integration



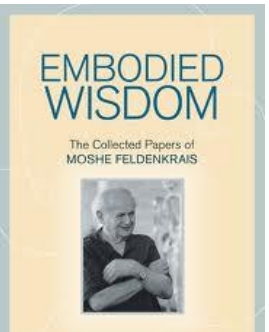


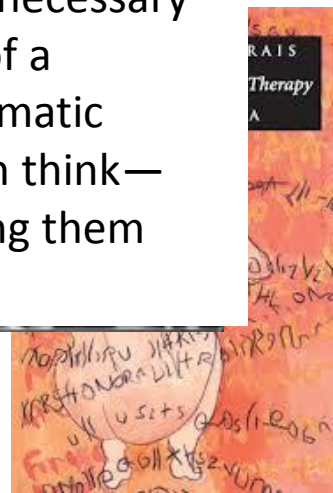
MF: "Without conscious attention to what one is feeling during an action and without applying the attention directly to the entire movement resulting from these actions, no development will occur—simple mechanical repetition will never make this come about."

Moshe Feldenkrais
(1904-1984)

Carl Ginsburg: “Learning itself is not conscious. The integration process itself is not conscious. Nevertheless the process depends on conscious processes in feeling and detecting changes. The consequence is felt as difference.”

MF: ”[T]he use and experience of the body are necessary to form mental functions. After the formation of a sufficient number of paths and patterns, the somatic support becomes less and less essential; we can think—that is, re-excite the formed patterns, regrouping them into new ones. “





Beringer, E. (Ed.). (2010). *Embodied wisdom: the collected papers of Moshe Feldenkrais*. Berkeley, CA: North Atlantic Books.

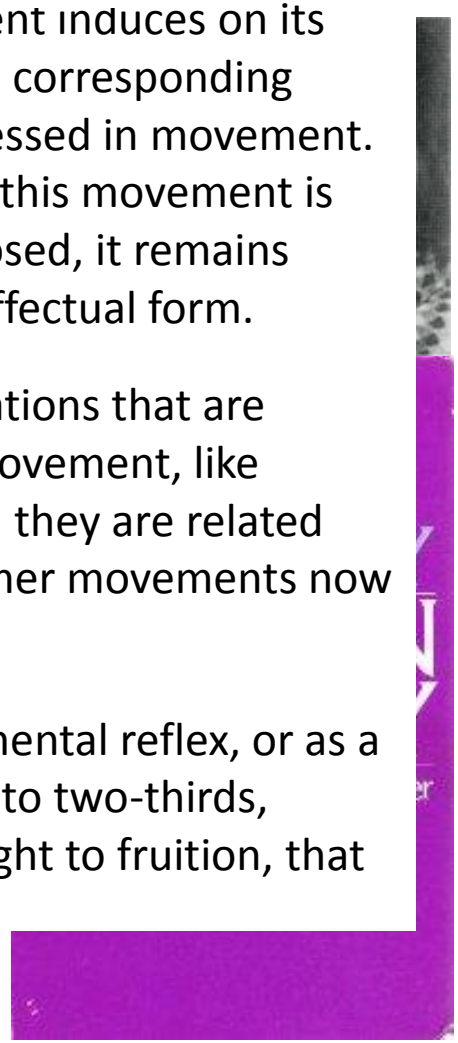
Mathematics as Inner Action

Lev Vygotsky
(1896-1934)

Every thought associated with movement induces on its own a certain preliminary straining of a corresponding muscular system that tends to be expressed in movement. If it remains only a thought, then since this movement is not brought to fruition and is not disclosed, it remains concealed in an entirely tangible and effectual form.

Even the most abstract thoughts of relations that are difficult to convey in the language of movement, like various mathematical formulas…., even they are related ultimately to particular residues of former movements now reproduced anew.

[T]hought [is] the first two-thirds of a mental reflex, or as a reflex that has been broken down up into two-thirds, having in mind a reflex that is not brought to fruition, that is inhibited in its external aspect.



Vygotsky, L. (1926/1997). *Educational psychology*. Boca Raton, FL: CRC.

Summary of Theoretical Introduction: Implications of Kinesiology for Embodied Learning

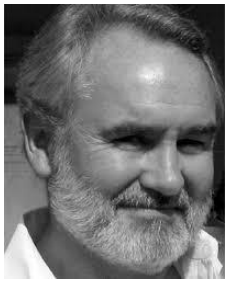
Thelen

Feldenkrais

Bernstein

Vygotsky

- Motor learning is a process of problem solving
- Motor learning creates refinement or reorganization of neural connections
- Motor dexterity is the sum total of automatisms the agent learns to enact so as to optimize performance given unexpected constraints
- Motor dexterity develops via the agent's explorative interaction in material ecology, sometimes via social intervention
- Motor competence cannot be taught directly – we can only create ecologies – fields of promoted action – wherein we govern constraints
- Motor learning emerges – body as vanguard, adapting to unexpected changes, discovering affordances; post facto consciousness to differences
- New coordination patterns as cognitive vehicles of reasoning



Conclusions

(a) From a systems perspective, learning is the development of new coordination dynamics among student, task, and environment

(b) Students develop sensorimotor schemes as their pragmatic solutions to designed interaction problems

(c) The 'sensori-' component of the scheme is a new attentional anchor, i.e., a real or imagined object, area, or other aspect or behavior of the perceptual manifold that emerges to facilitate motor coordination; and

(d) Introducing symbolic artifacts both elicits new affordances for students' motor control of the environment and, in so doing, shifts students' discourse into explicit mathematical re-visualization of the environment.

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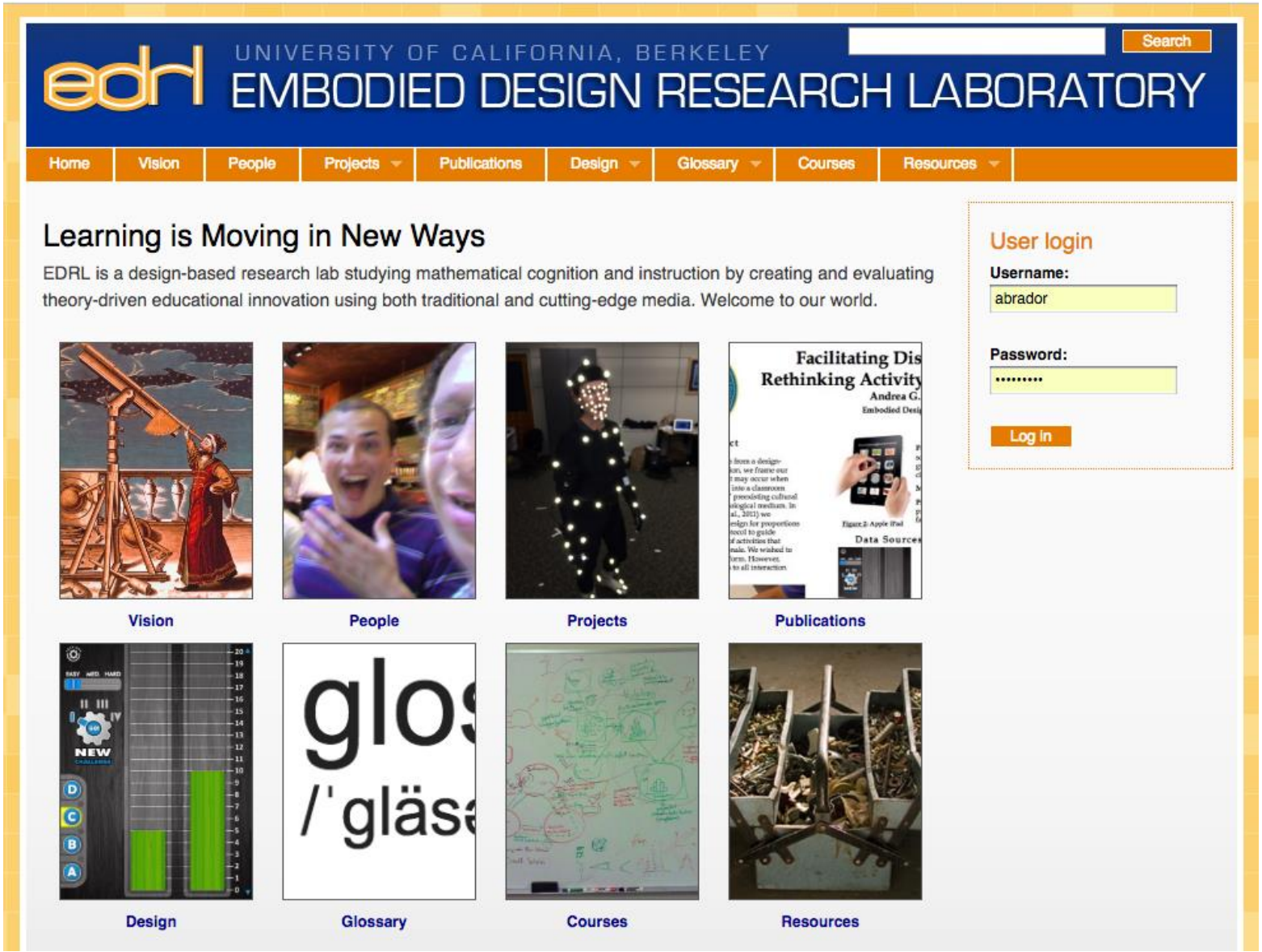

Investigative Approach: Design-Based Research

Conjecture-driven,

iterative,

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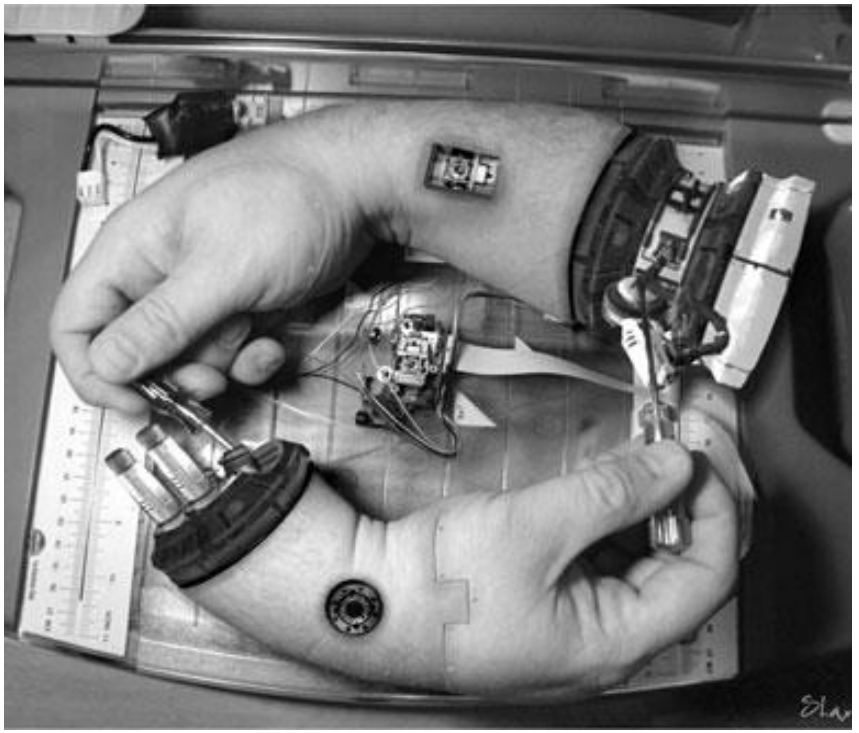
theory and design

co-inscribe

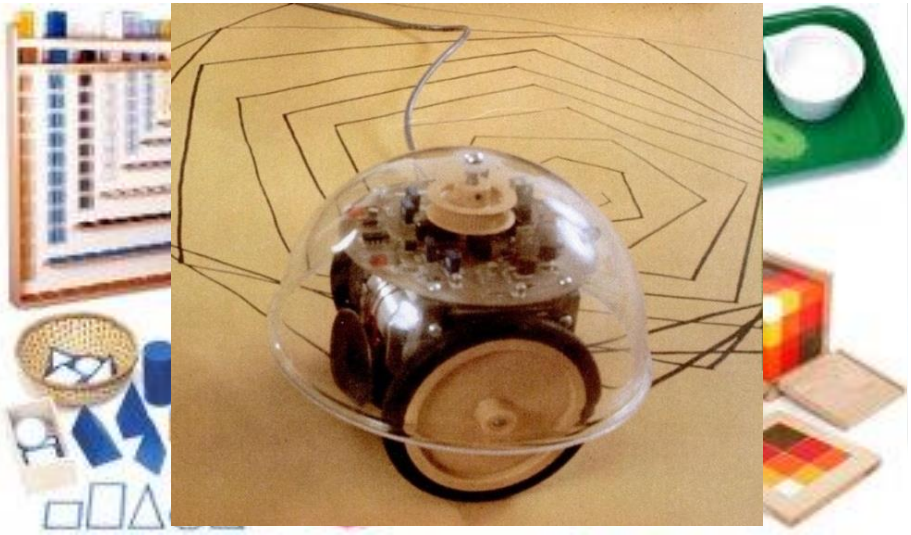


Abrahamson, D. (2009). Embodied design: Constructing means for constructing meaning. *Educational Studies in Mathematics, 70*(1), 27-47

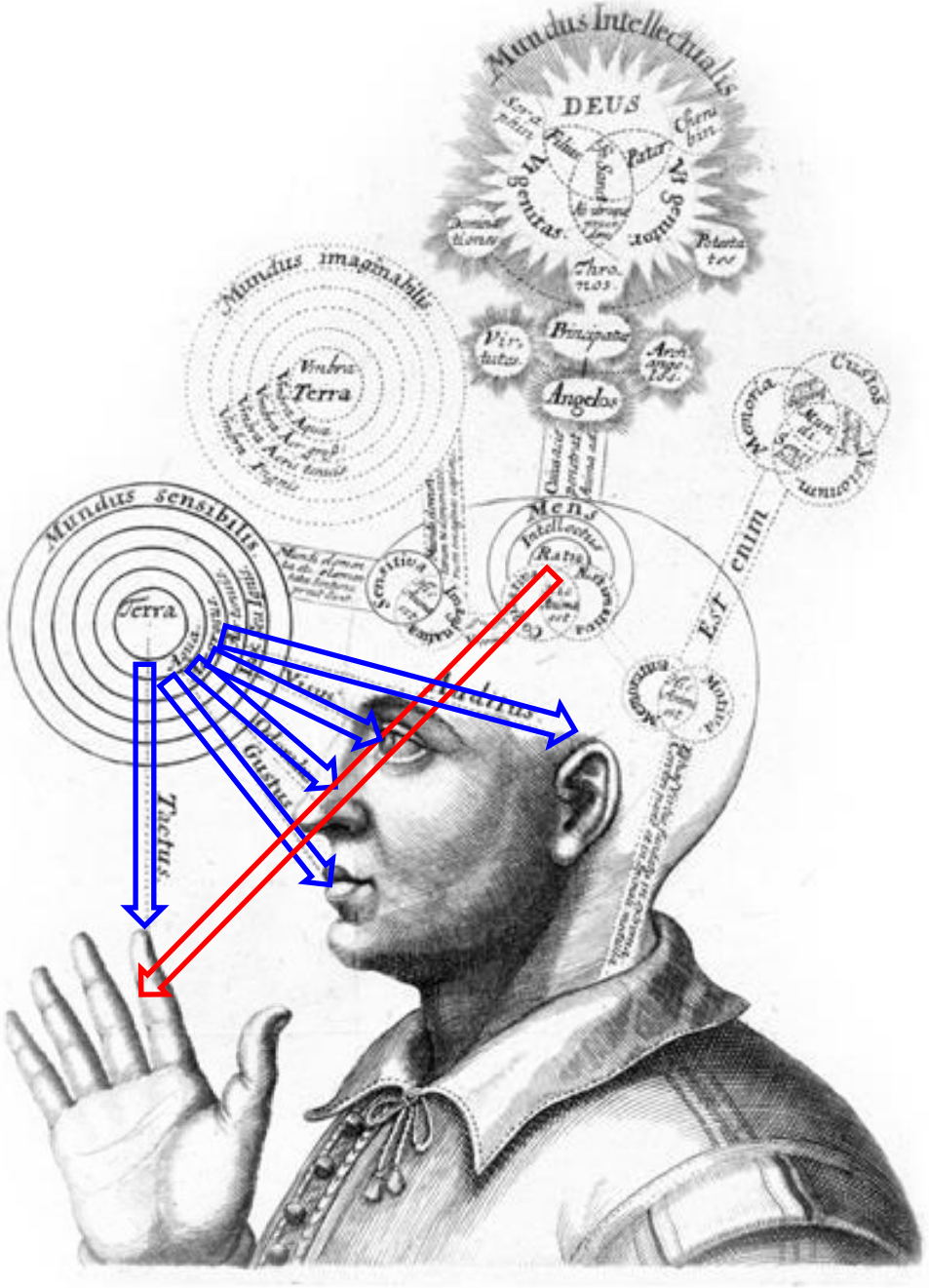
Embodied Design:

Constructing Means for Constructing Meaning

? Experience first, analyze later:
Building mathematical concepts from
perception, action, aesthetics, and common sense



- Friedrich Fröbel (b.1782)
- Maria Montessori (b. 1870)
- Hans Freudenthal (b. 1905)
- Caleb Gattegno (b. 1911)
- Zoltán Diénès (b. 1916)
- Seymour Papert (b. 1928)


Mundus Intellectualis
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Angelos
Mundus sensibilis
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Mens
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Embodied Mathematical Reasoning

Terrence **Tao**, MathOverflow